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Vol. XXXII No. 10 October 2014

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editorial

Mathematics, Mathematicians and Metamathematicians

It is very difficult to separate mathematics and metamathematics. Mathematicians are also philosophers. Indian mathematics is very closely related to astronomy. Newton's astronomy is not very much different from that of the Greeks and Indian astronomy. Initially expertise in mathematics was ability in numerical calculations. Geometry and calculus were also included in mathematics together with trigonometry. Modern mathematicians are not different from physicists. In quantum mechanics, crystallography and astronomy, one has to make a lot of calculations and as in the case of crystallography and the study of molecular spectra, a lot of calculations are needed. It is not just-calculation according to a given method but also discovering methods of calculations. Repeated and tedious calculations can be handled by modern computers available in big centres of research. But what is calculated is decided by the research scientists.

Great mathematicians such as Dirac and Einstein are not only physicists but also philosophers. Confining ourselves to modern physics, length, mass and time are considered to be constants for a particular body even in motion. In motion time changes but time is absolute unrelated to the energy in classical physics. According to Einstein's theory, length, mass and time are properties depending on their velocity or energy. Another example is "Dirac's Vacuum" which consists of all particles and anti-particles. In nuclear reactions particles and antiparticles are created out of this "vacuum". This is not Soonya of Buddhism. Rather it is Poorna of Upanishads.

Well, modern Mathematics, Physics and Philosophy are not different.

There is a saying that to eat sugarcane one need not get paid. Learning maths is enjoyment. Best of luck to our students.

Anil Ahlawat
Editor

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MATHS MUSING

Maths Musing was started in January 2003 issue of Mathematics Today with the suggestion of Shri Mahabir Singh. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material. During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.
Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

Prof. Dr. Ramanaiah Gundala, Former Dean of Science and Humanities, Anna University, Chennai

PROBLEM
Set 142

JEE MAIN

1. Let $f(x) = x^4 + ax^3 + bx^2 + cx + d$. If $f(2) = 1$, $f(3) = 2, f(4) = 3, f(5) = 4$, then $a + b + c + d =$
(a) 21 (b) 22 (c) 23 (d) 24

2. Let S be the sum of all 6-digit numbers formed by the six digits 1, 2, 2, 3, 3, 3. The sum of the digits of S is
(a) 25 (b) 30 (c) 32 (d) 35

3. If the complex number $z \neq 0, 1$, then the area of the quadrilateral with vertices $z, \bar{z}, \frac{1}{z}, \frac{1}{\bar{z}}$ is
(a) $\frac{1}{4}|z^2 - z^{-2}|$ (b) $\frac{|z|^4 + 1}{4|z|^2}$
(c) $\frac{1}{4}\frac{||z|^4 - 1|}{|z|^2}$ (d) $\frac{1}{4}\left|1 - \frac{1}{|z|^4}\right||z^2 - \bar{z}^2|$

4. The number of 3×3 symmetric matrices with five entries 0 and four entries 1 is
(a) 6 (b) 9 (c) 12 (d) 15

5. If $0 < A < B < \pi$, $\sin A + \sin B = \sqrt{\frac{3}{2}}$, $\cos A + \cos B = \frac{1}{\sqrt{2}}$, then $A + B =$
(a) 105° (b) 120° (c) 135° (d) 150°

JEE ADVANCED

6. If $n = 2015$, then $2015^n - 1988^n - 1978^n + 1951^n$ is divisible by
(a) 1998 (b) 1983 (c) 3966 (d) 2014

COMPREHENSION

Let L_1 be the line $2x - 2y + 3z - 2 = 0 = x - y + z + 1$ and L_2 be the line $x + 2y - z - 3 = 3x - y + 2z - 1 = 0$

7. The distance of the origin from the plane through L_1 and L_2 is
(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$ (c) $\frac{1}{3\sqrt{2}}$ (d) $\frac{1}{\sqrt{15}}$

8. L_1 and L_2 meet at a point P whose distance from the origin is
(a) $\sqrt{33}$ (b) $\sqrt{22}$ (c) $\sqrt{25}$ (d) $2\sqrt{41}$

INTEGER MATCH

9. In a triangle ABC, $a = 3$, $b = 2c$. The maximum area of the triangle is

MATCHING LIST

10. Column-I gives functions and Column-II gives their nature

	Column-I		Column-II
P.	$f: [0, \infty) \to [0, \infty)$, $f(x) = \frac{x}{1+x}$	1.	One-one onto
Q.	$f: R - \{0\} \to R$, $f(x) = x - \frac{1}{x}$	2.	One-one, not onto
R.	$f: R - \{0\} \to R$, $f(x) = x + \frac{1}{x}$	3.	Onto, not one-one
S.	$f: R \to R, f(x) = 2x + \sin x$	4.	Neither one-one nor onto

	P	Q	R	S
(a)	4	1	2	3
(b)	1	2	3	4
(c)	2	3	4	1
(d)	3	4	1	2

See Solution set of Maths Musing 141 on page no. 79.

Prof. Ramanaiah is the author of MTG JEE(Main & Advanced) Mathematics series

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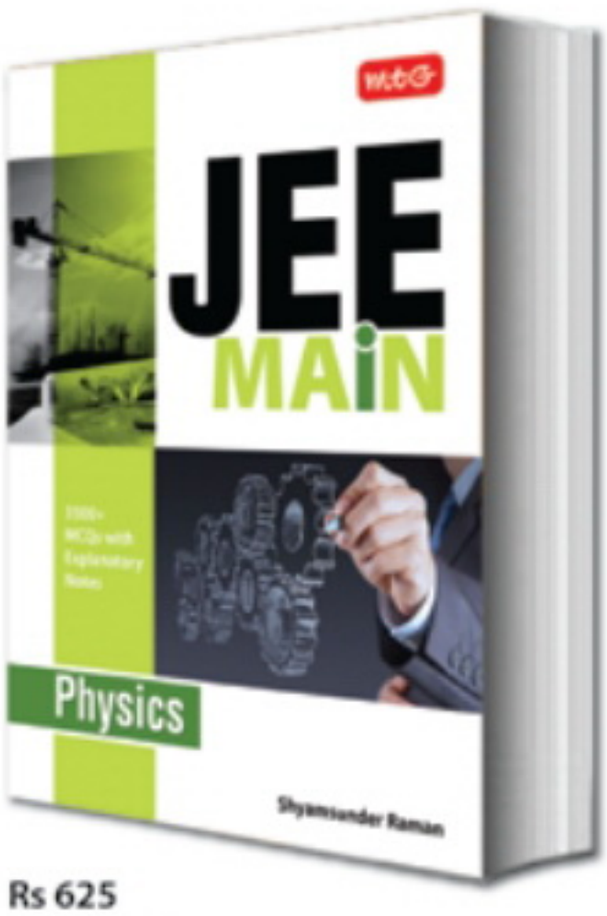

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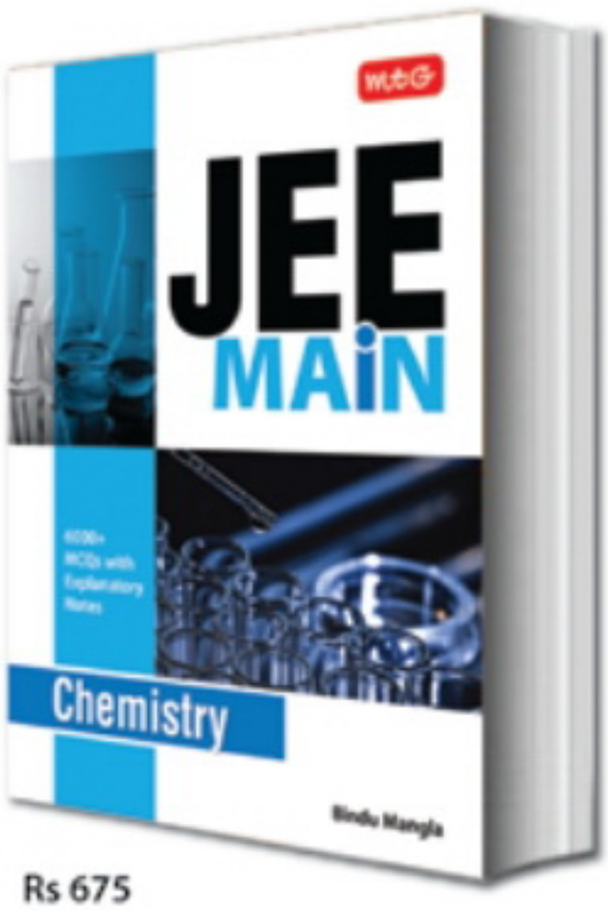

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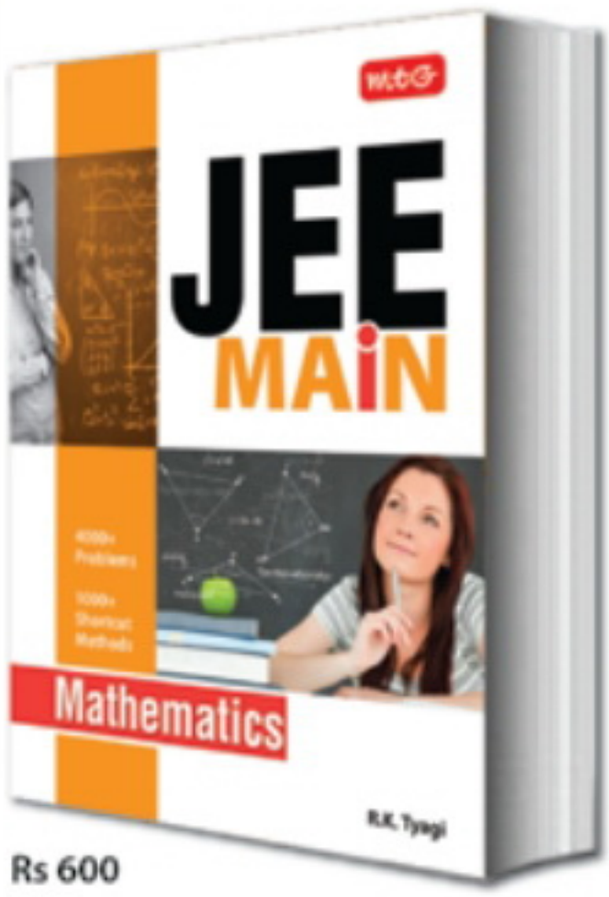

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ENRICH YOUR CONCEPTS ON XI

Permutations and Combinations, Binomial Theorem, Sequences and Series

Based on NCERT Pattern

Series-5

PERMUTATIONS AND COMBINATIONS

COUNTING PRINCIPLES

There are two fundamental counting principles

1. Multiplication principle
2. Addition principles

S.No.	Name	Definition
1.	Multiplication Principle	If one operation can be performed in m different ways and if corresponding to each way of performing this operation, there are n different ways of performing second operation, the two operations together can be performed in $m \times n$ ways.
2.	Addition Principle	If there are two operations such that they can be performed independently in m and n ways respectively, then either of the two operations can be be performed in $(m + n)$ ways.

FACTORIAL NOTATION

The continued product of first n natural numbers is called factorial n or n factorial and is denoted by $n!$ or $\lfloor\underline{n}$.

Thus, $n!$ or $\lfloor\underline{n} = 1 \cdot 2 \cdot 3 \cdot 4 \ldots (n-1) \cdot n$

$= n(n-1)(n-2) \ldots 3 \cdot 2 \cdot 1$ (in reverse order)

and we define $\lfloor\underline{0} = 1$

PERMUTATIONS

The different arrangements which can be made by taking some or all of the given things or object at a time is called permutation.

The total number of permutations of n distinct objects taking $r (0 \le r \le n)$ at a time is denoted by $P(n, r)$ or ${}^nP_r$ and defined by ${}^nP_r = \dfrac{n!}{(n-r)!}$

IMPORTANT THEOREM

- The number of permutations of n different objects taken r at a time where repetition is allowed is n^r.
- The number of permutations of n objects, where p objects are of the same kind and rest are all different $= \dfrac{n!}{p!}$.

  In fact, this theorem can be generalised to :

  The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of second kind, ..., p_k are of k^{th} kind and the rest, if any, are of different kind is $\dfrac{n!}{p_1!\, p_2! \ldots p_k!}$

CIRCULAR ARRANGEMENT

- The number of circular permutations of n different things taken all at a time is $(n-1)!$, if clockwise and anticlockwise orders are taken as different.
- The number of circular permutations of n different things taken all at a time is $\dfrac{1}{2}(n-1)!$, if clockwise and anticlockwise orders are taken as not different.
- Number of circular permutations of n different things taken r at a time. If clockwise and anticlockwise orders are taken as different, then

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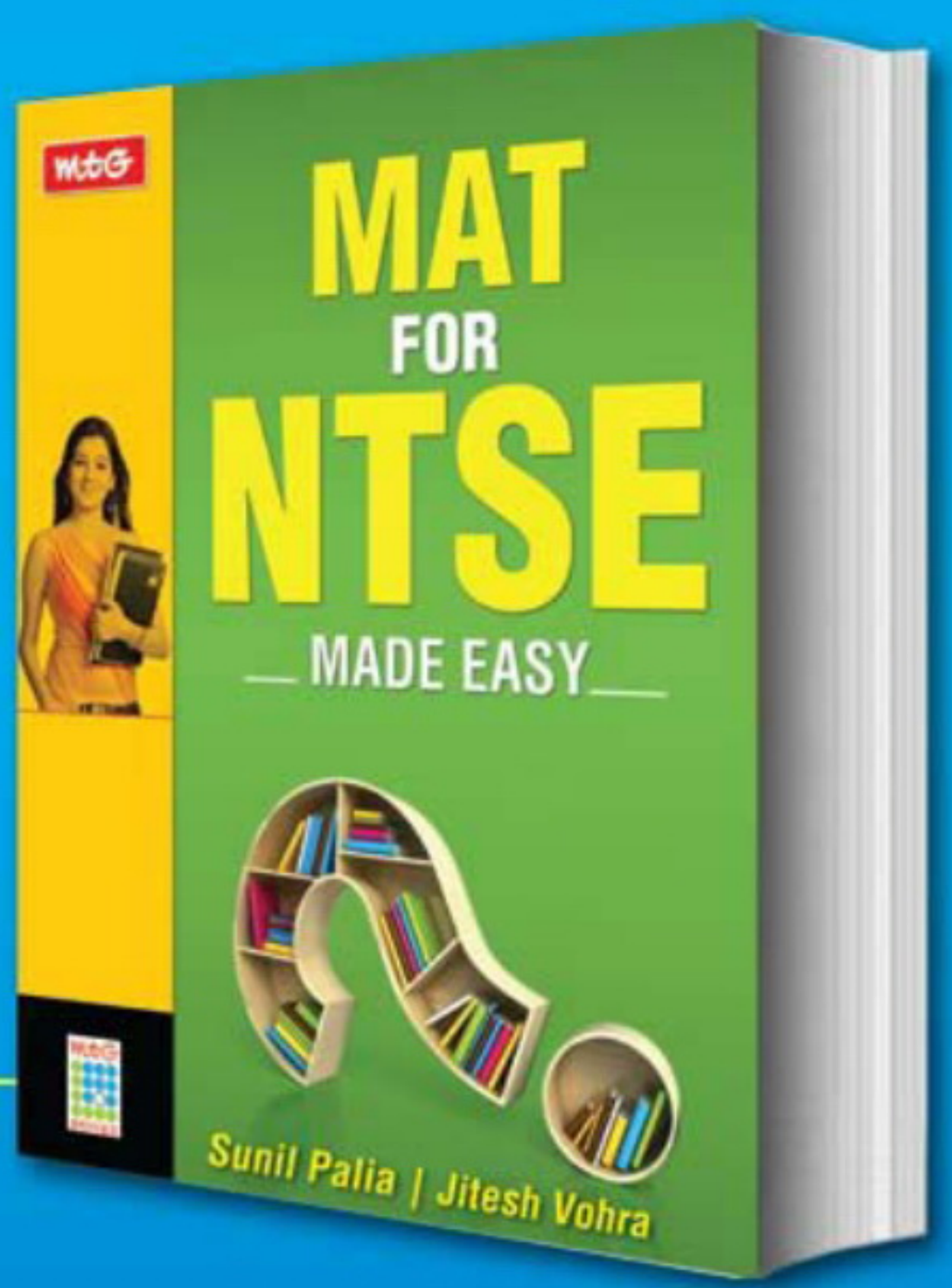

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the required number of circular permutations $= ({}^nP_r)/r$.

- Number of circular permutations of n different things taken r at a time. If clockwise and anticlockwise orders are taken as not different, then the required number of circular permutations $= ({}^nP_r)/(2r)$.

COMBINATIONS

Each of the different groups or selection which can be made by taking some or all of a number of objects, irrespective of their arrangement is called combination (combination means selection of objects).

The number of selections (combinations or groups) that can be formed from n different objects taken $r(0 \le r \le n)$ at a time is given by

$${}^nC_r = \frac{n}{r}\,{}^{n-1}C_{r-1} = \frac{n!}{(n-r)!r!}$$

DIFFERENCE BETWEEN PERMUTATIONS AND COMBINATIONS

In combination, only a group is made and the order of the objects are arranged is meaningless (immaterial). In permutation, only a group is formed, but also an arrangement of the objects in definite order is considered.

	Permutations	Combinations
1.	Problems of arrangements	Problems on selections, choose
2.	Problems of starting in a line, seated in a row	Problems on distributed group is formed
3.	Problems on digits	Problems of committee
4.	Problems on letters from a word	Problems of geometry

BINOMIAL THEOREM

The expansion of binomial theorem to any positive integral index is

$$(a+b)^n = {}^nC_0\,a^n + {}^nC_1\,a^{n-1}\,b + {}^nC_2\,a^{n-2}\,b^2 + \ldots + {}^nC_{n-1}\,ab^{n-1} + {}^nC_n\,b^n$$

The coefficients of ${}^nC_r$ in the binomial theorem are known as *binomial coefficients*.

Remarks :

1. There are $(n + 1)$ terms in the expansion of $(a + b)^n$, *i.e.* one more than the index.
2. The sum of the indicies of a and b is n in every term of the expansion.

GENERAL AND MIDDLE TERMS

1. General Term:

The $(r + 1)^{th}$ term is called the general term of the expansion $(a + b)^n$ and is denoted by T_{r+1},

i.e. $T_{r+1} = {}^nC_r\,a^{n-r}\,b^r$

2. Middle term:

(i) If the index n is even then there is only one middle term in the expansion of $(x + a)^n$ *i.e.*, $\left(\frac{n}{2}+1\right)^{th}$ term.

(ii) If the index n is odd, then there are two middle terms in the expansion of $(x + a)^n$ *i.e.*, $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+3}{2}\right)^{th}$

SEQUENCES AND SERIES

SEQUENCES

Let N denotes the set of all natural numbers and X be any non-empty set. A sequence or a progression is a mapping.

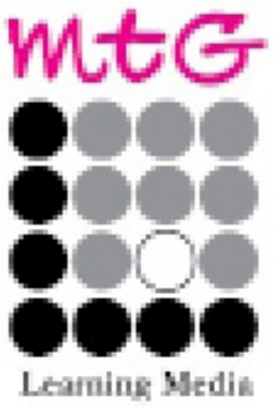
$f: N \to X$ given by $f(n) = x_n$ for all $n \in N$

We denote the sequence by (x_n) or $\{x_1, x_2, x_3, \ldots.\}$. The elements $x_n \in X$ are called terms of the sequence.

If $X = R$, we have a sequence of real numbers. If $X = C$, we have a sequence of complex numbers.

Finite and Infinite sequences:

A sequence is said to be finite if it has finite number of terms and if number of terms are infinite, then it will be infinite sequence.



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SERIES

An expression of the form $x_1 + x_2 + x_3 + ...$, where $x_1, x_2, x_3, ...$ is a sequence of numbers, is called **series.** If the number of terms in the series is finite, it is called a **finite series,** otherwise it is called an **infinite series**

we write $x_1 + x_2 + x_3 + ... + x_n + ...\infty = \sum_{i=1}^{\infty} x_i$

ARITHMETIC PROGRESSION

A sequence (finite or infinite) is called an Arithmetic Progression (A.P.) iff the difference of any term from its preceding term is constant.

i.e., $a_1, a_2, a_3, ..., a_n, ...$ is an A.P. if $a_{n+1} = a_n + d$, $n \in N$, where a_1 is called the first term and the constant term d is called the common difference of the A.P.

- General term of A.P. is $a_n = a + (n - 1)\, d$
  Thus, if a is the first term and d is the common difference, then A.P. is
  $a, a + d, a + 2d, ..., a + (n - 1)\, d$
  or, $a, a + d, a + 2d, ...$ according as it is finite or infinite.
- If the last term of an A.P. consisting of n terms is denoted by l, then $l = a + (n - 1)d$.
- The sum of n terms of A.P. is

$$S_n = \frac{n}{2}[2a + (n-1)d] \text{ or, } S_n = \frac{n}{2}[a + l]$$

PROPERTIES OF AN A.P.

(i) If a constant is added to each term of an A.P., then the resulting sequence is also an A.P.

(ii) If a constant is subtracted from each term of an A.P., then the resulting sequence is also an A.P.

(iii) If each term of an A.P. is multiplied by a constant, then the resulting sequence is also an A.P.

(iv) If each term of an A.P. is divided by a non-zero constant, then the resulting sequence is also an A.P.

ARTHMETIC MEAN (A.M.)

(i) An A.M. between 2 numbers

Let a, b be the two given numbers and A be the A.M. between them, then

a, A, b are in $A.P. \Rightarrow A = \frac{a+b}{2}$

(ii) n A.M.'s between two numbers

Let a, b be the given numbers and $A_1, A_2, ..., A_n$ be n A.M.'s between them, then

$$\text{A.M.} = n\left(\frac{a+b}{2}\right)$$

Remarks :

(i) Three numbers in A.P. be taken as $a - d$, a, $a + d$

(ii) Four numbers in A.P. be taken as $a - 3d$, $a - d$, $a + d$, $a + 3d$

(iii) Five numbers in A.P. be taken as $a - 2d$, $a - d$, a, $a + d$, $a + 2d$

GEOMETRIC PROGRESSION (G.P.)

A sequence (finite or infinite) of non-zero terms is called a G.P. iff the ratio of any term to its preceding term is constant.

i.e. $a_1, a_2, a_3, ..., a_n, ...$ is in G.P. iff $\frac{a_{k+1}}{a_k} = r, \ k \geq 1$

where a_1 is the first term and r is called the common ratio.

- General term of a G.P. : $T_n = ar^{n-1}$
- Last term of a G.P. consisting of n terms : $l = ar^{n-1}$
- Sum to n terms of a G.P. :

$$S_n = \frac{a(1-r^n)}{1-r}; \ r<1 \text{ or } S_n = \frac{a(r^n-1)}{r-1}; \ r>1$$

PROPERTIES OF G.P.

(i) If all the terms of a G.P. be multiplied or divided by a non-zero constant, then it remains a G.P. with the same common ratio.

(ii) The reciprocals of the terms of a given G.P. form a G.P.

(iii) If each term of a G.P. be raised to the same power, the resulting sequence also forms a G.P.

(iv) In a finite G.P., the product of the terms equidistant from the beginning and the end is always same and is equal to the product of the first and the last term.

(v) Three non-zero numbers a, b, c are in G.P. iff $b^2 = ac$.

GEOMETRIC MEAN

(i) **An G.M. between two numbers**

Let a and b two positive numbers, then G.M. $= \sqrt{ab}$.

(ii) ***n* G.M's between two numbers**

Let a and b two positive numbers, and G_1, G_2, .., G_n be n G.M's between them, then G.M. $= \left(\sqrt[n]{ab}\right)$

Remarks :

(i) Three numbers in G.P. can be taken as $\frac{a}{r}, a, ar$

(ii) Four numbers in G.P. can be taken as $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

(iii) Five numbers in G.P. can be taken as $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$.

RELATIONSHIP BETWEEN A.M. AND G.M.

Let A and G be A.M. and G.M. of two positive real numbers a and b, then

$$A = \frac{a+b}{2} \quad \text{and} \quad G = \sqrt{ab}$$

$$A - G = \frac{a+b}{2} - \sqrt{ab} = \frac{\left(\sqrt{a} - \sqrt{b}\right)^2}{2} \geq 0$$

i.e. $A \geq G$

SUM TO *n* TERM OF SPECIAL SERIES

(i) $S_n = 1 + 2 + 3 + ...n = \frac{n(n+1)}{2}$

(ii) $S_n = 1^2 + 2^2 + 3^2 + ...+ n^2 = \frac{n(n+1)(2n+1)}{6}$

(iii) $S_n = 1^3 + 2^3 + 3^3 + ... + n^3 = \frac{n^2(n+1)^2}{4} = \frac{[n(n+1)]^3}{4}$

VERY SHORT ANSWER TYPE (1 MARK)

1. Find the n^{th} term and 12^{th} term of the sequence $2i, -6i, 18i, -54i, \ldots$.
2. In the binomial expansion of $(x + y)^n$, the coefficients of 4^{th} and 13^{th} terms are equal. Find n.
3. How many different arrangements can be made from the letters of the word "ENGINEERING"?
4. Find n, if ${}^nP_4 : {}^{n-1}P_3 = 9 : 1$.
5. If $(1 + x)^n = C_0 + C_1 x + C_2 x^2 + ...+ C_n x^n$, then prove that $\frac{C_1}{C_0} + \frac{2C_2}{C_1} + \frac{3C_3}{C_2} + ... + \frac{nC_n}{C_{n-1}} =$ Sum of first n natural numbers.

SHORT ANSWER TYPE (4 MARKS)

6. Find the coefficient of x^4 in the expansion of $(1 + x - 2x^2)^6$.
7. The 12^{th} term of a series in A.P. is -13 and the sum of first four terms of it is 24. Find the sum of its first 10 terms.
8. How many numbers of five digits can be made if the repetition of digits allowed?
9. Sum to n terms : $4 + 44 + 444 + ...$
10. The digit of a positive integer having three digits are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.

LONG ANSWER TYPE (6 MARKS)

11. If the coefficients of p^{th}, $(p + 1)^{th}$ and $(p + 2)^{th}$ terms in the expansion of $(1 + x)^n$ are in A.P., show that $n^2 - n(4p + 1) + 4p^2 - 2 = 0$.
12. How many numbers each lying between 100 and 1000 can be formed with the digits 2, 0, 3, 4, 5, no digit being repeated? How many of these are odd?
13. Find the number of all possible arrangements of the letters of the word "MATHEMATICS" taken four at a time.
14. Find the term independent of x in the expansion of $(1 + x + 2x^3)\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$.
15. If a, b, c are in G.P. and x, y are A.M. of a, b and b, c respectively. Show that $\frac{a}{x} + \frac{c}{y} = 2$; $\frac{1}{x} + \frac{1}{y} = \frac{2}{b}$.

SOLUTIONS

1. The given sequence is a G.P. with $a = 2$ and $r = -3$.

$\therefore \quad T_n = ar^{n-1} = (2\,i)\,(-3)^{n-1}$

$= (-1)^{n-1} \times 2 \times 3^{n-1}\, i$

For the 12^{th} term, putting $n = 12$, we get

$T_{12} = (-1)^{11} \times 2 \times 3^{11}\, i = -2 \times 3^{11}\, i.$

2. In the expansion of $(x + y)^n$,

$T_4 = T_{3+1} = {}^nC_3\, x^{n-3}\, y^3$ and

$T_{13} = T_{12+1} = {}^nC_{12}\, x^{n-12}\, y^{12}$

We are given that

${}^nC_3 = {}^nC_{12} \implies 12 = n - 3 \implies n = 15$

If ${}^nC_r = {}^nC_{n-r}$, then $r = n - r$

3. The word 'ENGINEERING' has 11 letters of which 3 are E's; 3 are N's; 2 are G's and 2 are I's and the remaining one is different.

Hence, the required number of permutations

$$\frac{\lfloor 11}{\lfloor 3 \lfloor 3 \lfloor 2 \lfloor 2} = 277200.$$

4. We have,

${}^nP_4 = n(n-1)\,(n-2)\,(n-3)$ and

${}^{n-1}P_3 = (n-1)\,(n-2)\,(n-3).$

But ${}^nP_4 : {}^{n-1}P_3 = 9 : 1$

$$\implies \frac{n(n-1)\,(n-2)\,(n-3)}{(n-1)\,(n-2)\,(n-3)} = \frac{9}{1} \implies n = 9$$

5. L.H.S. $= \frac{C_1}{C_0} + \frac{2C_2}{C_1} + \frac{3C_3}{C_2} + \ldots$

$$+ \frac{nC_n}{C_{n-1}} = \frac{n}{1} + \frac{2\frac{n(n-1)}{2\times 1}}{n} + \frac{3\frac{n(n-1)\,(n-2)}{3\times 2\times 1}}{\frac{n(n-1)}{2\times 1}} + \ldots + \frac{n\times 1}{n}$$

$= n + (n-1) + (n-2) + \ldots + 1 = 1 + 2 + 3 + \ldots + (n-1) + n$ as desired.

6. $(1 + x - 2x^2)^6 = [1 + (x - 2x^2)]^6$

$= 1 + {}^6C_1\,(x - 2x^2)^1 + {}^6C_2\,(x - 2x^2)^2 + {}^6C_3\,(x - 2x^2)^3 + {}^6C_4 \cdot (x - 2x^2)^4 + {}^6C_5 \cdot (x - 2x^2)^5 + (x - 2x^2)^6.$

The first, second, sixth and seventh terms do not contain x^4.

$\therefore$ The coefficient of x^4 in the above expansion

$= 1 \times {}^6C_2 \times 2^2 + {}^6C_3\,(3)\,(-2) + {}^6C_4 \times 1$

$$= \frac{6\times 5}{2\times 1} \times 4 + \frac{6\times 5\times 4}{3\times 2\times 1} \times (-6) + \frac{6\times 5}{2\times 1}$$

$= 60 - 120 + 15 = -45.$

7. Let a = first term of A.P. and d = common difference of A.P.

$\therefore \quad 12^{th}$ term $= a + (12-1)\,d = a + 11\,d = -13$ ...(i)

Sum of first 4 terms $= S_4 = \frac{4}{2}\left[2a + (4-1)\,d\right]$

$\implies \quad 2\,(2a + 3d) = 24$ ...(ii)

Solving (i) & (ii), we get $a = 9,\ d = -2$.

$\therefore$ Sum of first ten terms

$$S_{10} = \frac{10}{2}\left[2\times 9 + (10-1)(-2)\right] = 5\,(18 - 18) = 0$$

8. The first place can be filled by any of the digits from 1, 2, 3, 4, 5, ..., 9 i.e. in 9 ways. Each of second, third, fourth and fifth place can be filled by any of the digits from 0, 1, 2, ..., 9 *i.e.* in 10 ways.

$\therefore$ Total number of five digit numbers is

$9 \times 10 \times 10 \times 10 \times 10 = 90000.$

9. $4 + 44 + 444 + \ldots\ldots$ to n terms

$= 4\,[1 + 11 + 111 + \ldots\ldots$ to n terms$]$

$= \frac{4}{9}\,[9 + 99 + 999 + \ldots\ldots$ to n terms$]$

$= \frac{4}{9}\,[(10 - 1) + (10^2 - 1) + (10^3 - 1) + \ldots\ldots$ to n terms$]$

$= \frac{4}{9}\,[(10 + 10^2 + 10^3 + \ldots\ldots$ to n terms$)$

$- (1 + 1 + 1 + \ldots\ldots$ to n terms$)]$

$$= \frac{4}{9}\left[\frac{10\cdot(10^n - 1)}{10 - 1} - n\right] = \frac{4}{9}\left[\frac{10}{9}\left(10^n - 1\right) - n\right]$$

10. Since the given number has three digits and the digits are in A.P., let the digits in the unit's place, ten's place and hundred's place in the original number be $a - d$, a, $a + d$ respectively.

We have, $(a - d) + a + (a + d) = 15 \implies a = 5.$

The original number :

$(5 - d) + 5 \times 10 + (5 + d) \times 100 = 99\,d + 555$

The number on reversing the digits :

$= (5 + d) + 5 \times 10 + (5 - d) \times 100$

$= -99d + 555.$

According to given, we have

$-99d + 555 = (99d + 555) - 594$

$\Rightarrow \quad -198d = -594 = 3.$

Hence, the digits in unit's place, ten's place and hundred's place in the original number are respectively $5 - 3, 5, 5 + 3$, *i.e.*, 2, 5, 8.

Therefore, the original number is 852.

11. The p^{th}, $(p + 1)^{th}$ and $(p + 2)^{th}$ terms in the expansion of $(1 + x)^n$ are

x^{p-1}, ${}^nC_p\, x^p$ and ${}^nC_{p+1}\, x^{p+1}$.

since coefficients of these three terms are in A.P.

$\therefore \quad {}^nC_{p-1} + {}^nC_{p+1} = 2\,{}^nC_p.$

$$\Rightarrow \quad \frac{n!}{(n-p+1)!\,(p-1)!} + \frac{n!}{(p+1)!\,(n-p-1)!} = 2 \cdot \frac{n}{p!\,(n-p)!}$$

$$\Rightarrow \quad \frac{1}{(n-p+1)(n-p)(n-p-1)!\,(p-1)!} + \frac{1}{(p+1)p(p-1)!\,(n-p-1)!} = \frac{2}{p(p-1)!\,(n-p)(n-p-1)!}$$

$$\Rightarrow \quad \frac{1}{(n-p+1)\,(n-p)} + \frac{1}{p\,(p+1)} = \frac{2}{(n-p)\,p}$$

$$\Rightarrow \quad \frac{(p+1)\,p + (n-p+1)\,(n-p)}{(n-p+1)\,(n-p)\,p\,(p+1)} = \frac{2}{(n-p)\,p}$$

$\Rightarrow \quad (n - p + 1)\,(n - p) + p\,(p + 1) = 2\,(n - p + 1)\,(p + 1)$

$\Rightarrow \quad n^2 + p^2 - 2np + n - p + p^2 + p = 2\,(np - p^2 - p + n + p + 1)$

$\Rightarrow \quad n^2 - 2np + n + 2p^2 = 2np - 2p^2 + 2n + 2.$

$\Rightarrow \quad n^2 - 4np - n + 4p^2 - 2 = 0$

$\Rightarrow \quad n^2 - n\,(4p + 1) + 4p^2 - 2 = 0.$

12. Number lying between 100 and 1000 consist of 3 digits only.

Now, Total number of 3 digits numbers which can be formed by taking 3 at a time from the given 5 digits (2, 0, 3, 4, 5) is ${}^5P_3 = 60$.

But this also includes those which begin with 0, the number of which is ${}^4P_2 = 12$ [once '0' is fixed in hundred's place, remaining two places can be filled by available 4 digits in ${}^4P_2$ ways]

$\therefore$ The required number of 3 digit numbers $= 60 - 12 = 48.$

Now, to find how many of these are odd, we observe that such number must end with 3 or 5, but must not begin with 0.

The number of 3 digit numbers ending with 3 is ${}^4P_2 = 12$.

Out of these, number of those, beginning with 0 (*i.e.*, fixing 0 in 100's place and 3 in 1's place) is ${}^3P_1 = 3$.

$\therefore$ Number of 3 digit numbers ending with 3 and not beginning with 0 is $12 - 3 = 9$.

similarly, number of 3 digit numbers ending with 5 and not begining with 0 is also 9.

$\therefore$ Total number of odd numbers of 3 digits $= 9 + 9 = 18.$

13. The letters of the word 'MATHEMATICS' are M, M; T, T; A, A; H,E, C, I and S.

So there are eight distinct letters of which M, T and A are repeated twice. There are three possible cases.

Case I : When we use four distinct letters, the number of arrangements is

$${}^8P_4 = \frac{\lfloor 8}{\lfloor 8-4} = \frac{8\times7\times6\times5\times\lfloor 4}{\lfloor 4} = 1680$$

Case II : When a pair of repeated letters and two distinct letters are used, the number of arrangements

$$= {}^3C_1 \times {}^7C_2 \times \frac{\lfloor 4}{\lfloor 2} = 3\times21\times12 = 756.$$

(When a pair of repeated letters is selected which can be done in ${}^3C_1$ ways, we are left with seven letters of which 2 are to be selected and each selection of four letters can be arranged in $\lfloor 4 / \lfloor 2$ ways).

Case III : When two pairs of repeated letters are used, the number of arragngments

$$= {}^3C_2 \times \frac{\lfloor 4}{\lfloor 2 \lfloor 2} = 3 \times 6 = 18.$$

Hence, the total number of arrangements

$$= 1680 + 756 + 18 = 2454.$$

14. General term of the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$

$$T_{r+1} = {}^9C_r \left(\frac{3}{2}x^2\right)^{9-r} \left(-\frac{1}{3x}\right)^r$$

$$= {}^9C_r \left(\frac{3^{9-r}}{2^{9-r}}\right) x^{18-2r} \left(\frac{(-1)^r}{3^r\ x^r}\right)$$

$$= {}^9C_r (-1)^r \left(\frac{3^{9-2r}}{2^{9-r}}\right) x^{18-3r} \qquad ...(i)$$

In the product $(1 + x + 2x^3)\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$, the term independent of x

= 1 × term not containing x in

$$\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9 + x$$

$$x\left\{\text{term containing } \frac{1}{x} \text{ in } \left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9\right\}$$

$$+ 2x^3\left\{\text{term containing } x^{-3} \text{ in } \left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9\right\} \qquad ...(ii)$$

So, we need three terms in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$.

Term without x :

From (i), T_{r+1} will not contain x if $18 - 3r = 0$, *i.e.*, if $r = 6$ and this term is T_7

$$= T_7 = {}^9C_6 (-1)^6 \frac{3^{9-12}}{2^{9-6}} x^0 = {}^9C_3 \frac{1}{2^3 3^3}$$

$$= \frac{9 \times 8 \times 7}{1 \times 2 \times 3} \times \frac{1}{8 \times 27} = \frac{7}{18}.$$

Term containing $\frac{1}{x}$:

From (i), T_{r+1} will contain $\frac{1}{x}$ if $18 - 3r = -1$,

i.e., if $3r = 19$, *i.e.*, if $r = \frac{19}{3}$ which is not possible as r must be a whole number not exceeding 9.

So, there is no term with $\frac{1}{x}$ in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$.

Term containing x^{-3} :

From (i), T_{r+1} will contain x^{-3} if $18 - 3r = -3$, *i.e.*, $r = 7$ and this term is T_8

$$T_8 = {}^9C_7 (-1)^7 \frac{3^{9-14}}{2^{9-7}} x^{18-21} = {}^9C_7 (-1)^7 \frac{3^{-5}}{2^2} x^{-3}$$

$$= -\frac{9 \times 8}{1 \times 2} \frac{1}{3^5 \times 2^2} x^{-3} = -\frac{1}{27} x^{-3}.$$

Then from (ii), required term without x

$$= 1 \times \frac{7}{18} + 0 + 2x^3 \left(-\frac{1}{27}x^{-3}\right)$$

$$= \frac{7}{18} - \frac{2}{27} = \frac{21 - 4}{54} = \frac{17}{54}.$$

15. As a, b, c are in G.P. $\Rightarrow b^2 = ac$

$\because$ x, y are A.M. of a, b and b, c respectively

$\therefore$ $\frac{a+b}{2} = x$ and $\frac{b+c}{2} = y$

$\therefore$ $\frac{a}{x} + \frac{c}{y} = \frac{2a}{a+b} + \frac{2c}{b+c}$

$$= 2\left[\frac{ab + ac + ac + bc}{(a+b)(b+c)}\right]$$

$$= 2\frac{(ab + b^2 + ac + bc)}{(a+b)(b+c)} = 2\frac{\{b(a+b) + c(a+b)\}}{(a+b)(b+c)}$$

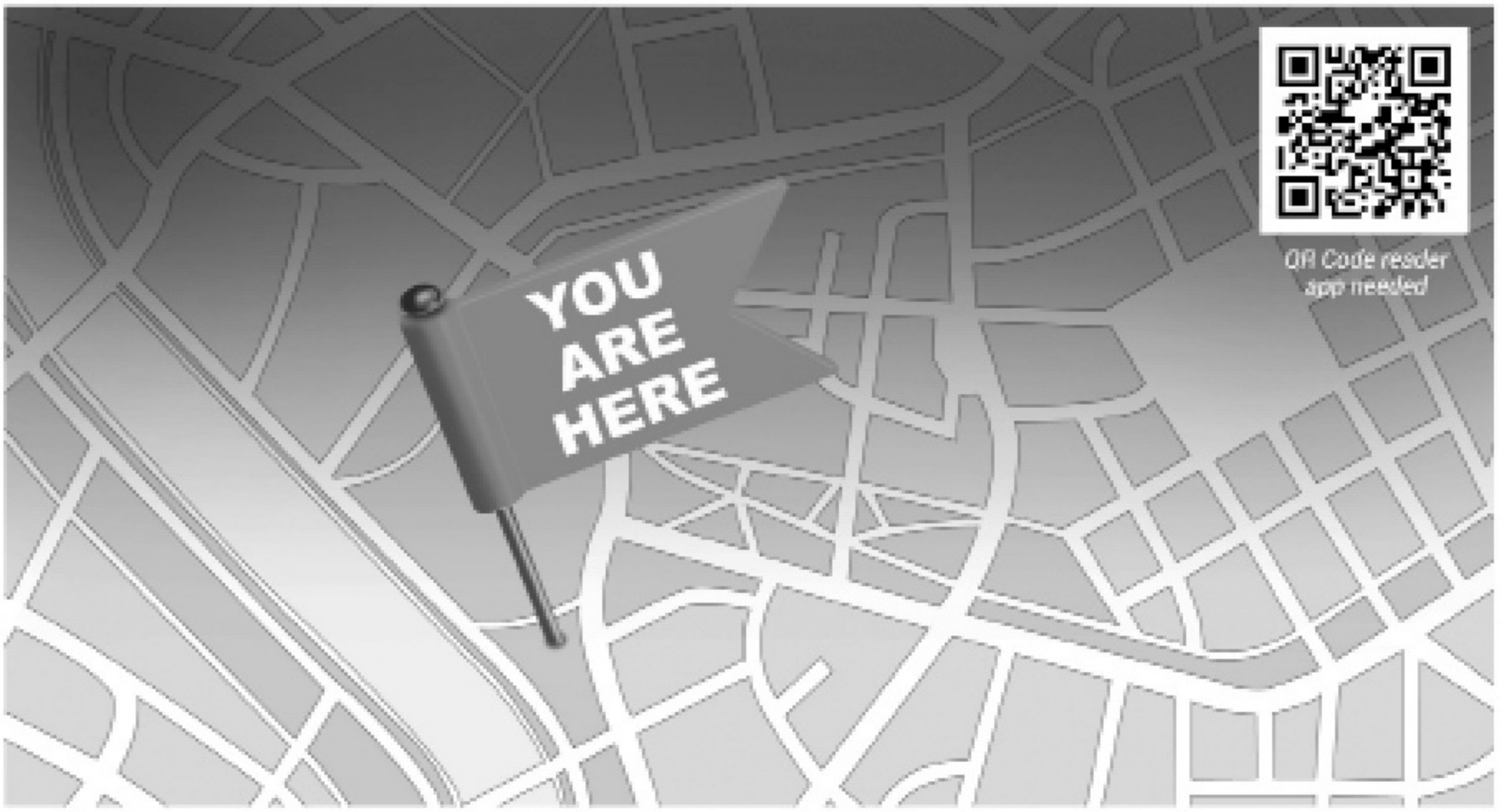
$$= 2\frac{(b+c)(a+b)}{(b+c)(a+b)} = 2 \qquad ...(i)$$

Again, $\frac{1}{x} + \frac{1}{y} = \frac{2}{a+b} + \frac{2}{b+c}$

$$= \frac{2(b+c+a+d)}{(a+b)(b+c)} = \frac{2(b^2 + bc + ab + ac)}{b(a+b)(b+c)}$$

$$= \frac{2\{b(a+b) + c(a+b)\}}{b(a+b)(b+c)} = \frac{2(a+b)(b+c)}{b(a+b)(b+c)} = \frac{2}{b} \qquad ...(ii)$$

from (i) & (ii), Hence Proved. ■■



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Math Archives

10 Best Problems

Math Archives, as the title itself suggests, is a collection of various challenging problems related to the topics of IIT-JEE Syllabus. This section is basically aimed at providing an extra insight and knowledge to the candidates preparing for IIT-JEE. In every issue of MT, challenging problems are offered with detailed solution. The readers' comments and suggestions regarding the problems and solutions offered are always welcome.

1. A bag contains 10 white and 3 black balls. Balls are drawn one by one without replacement till all the black balls are drawn. The probability that the procedure of drawing balls will come to an end at the seventh draw is

(a) $\frac{15}{286}$ (b) $\frac{105}{286}$ (c) $\frac{35}{286}$ (d) $\frac{7}{286}$

2. Let $f : R \to R$ be a differentiable function satisfying $f(y)f(x - y) = f(x)\ \forall\ x, y \in R$ and $f'(0) = p$, $f'(5) = q$, then $f'(-5)$ is

(a) $\frac{p^2}{q}$ (b) $\frac{p}{q}$ (c) $\frac{q}{p}$ (d) q

3. The sum of all divisors of the least natural number having 12 divisors is

(a) 168 (b) 188
(c) 156 (d) None of these

4. The number of ordered triplets (p, q, r) where $1 \le p, q, r \le 10$ such that $2^p + 3^q + 5^r$ is a multiple of 4, is $(p, q, r \in N)$

(a) 1000 (b) 500 (c) 250 (d) 125

5. A variable triangle is inscribed in a circle of radius R. If the rate of change of a side is R times the rate of change of the opposite angle, then that angle is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

6. The value of $\lim\limits_{n\to\infty}\left(\sqrt[3]{n^2 - n^3} + n\right)$ is

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

7. $ABCD$ and $PQRS$ are two variable rectangles such that A, B, C and D lie on PQ, QR, RS and SP respectively and perimeter 'x' of $ABCD$ is constant. If the maximum area of $PQRS$ is 32, then x is equal to

(a) 8 (b) 10 (c) 12 (d) 16

8. In ΔABC, least value of $\frac{e^A}{A} + \frac{e^B}{B} + \frac{e^C}{C}$ is equal to

(a) $\frac{\pi}{3}e^{\pi/3}$ (b) $\frac{\pi}{9}e^{\pi/3}$

(c) $\frac{9}{\pi}e^{\pi/3}$ (d) None of these

9. The probability that the birth days of six different persons will fall in exactly two calendar months is

(a) $\frac{341}{12^5}$ (b) $\frac{66}{12^5}$

(c) $\frac{352}{12^5}$ (d) None of these

10. The number of solutions of the equation $\sin^{-1}\left(\frac{1+x^2}{2x}\right) = \frac{\pi}{2}(\sec(x-1))$ is/are

(a) 1 (b) 3 (c) 2 (d) infinite

SOLUTIONS

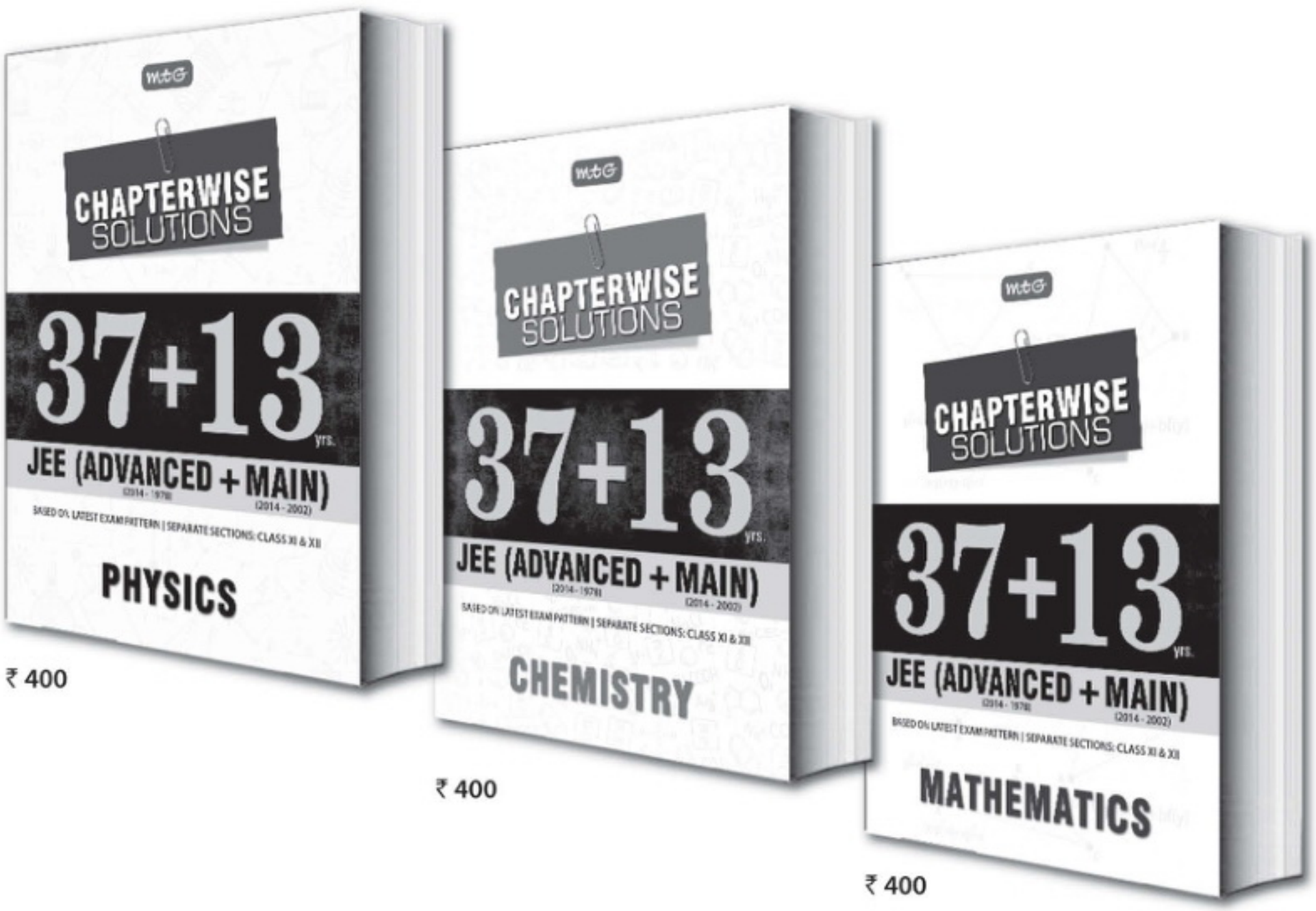
1. (a) : Required probability = Probability of getting exactly two black balls and 4 white balls in 1 to 6th draw × probability of getting 3rd black ball in 7th draw $= \frac{{}^3C_2 \cdot {}^{10}C_4}{{}^{13}C_6} \times \frac{1}{7} = \frac{15}{286}$.

2. (a) : Differentiating the functional equation with respect to x, we get $f(y)f'(x - y) = f'(x)$
Putting $x = y$, we get $f'(x) = pf(x)$

$$\int \frac{f'(x)}{f(x)}dx = \int p\,dx + C$$

By : **Prof. Shyam Bhushan,** Director, Narayana IIT Academy, Jamshedpur. Mob. : 09334870021

Mad about rehearsing?



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$\Rightarrow \log_e f(x) = px + C$

at $x = 0$, $f'(0) = pf(0) \Rightarrow f(0) = 1$

$\Rightarrow \log_e f(0) = C \Rightarrow C = 0$

$\Rightarrow f(x) = e^{px} \Rightarrow f'(x) = pe^{px}$

$$f'(5) = pe^{5p} = q \Rightarrow e^{5p} = \frac{q}{p}$$

$$f'(-5) = pe^{-5p} = \frac{p^2}{q}$$

3. (a) : $12 = 4 \times 3 = 2 \times 2 \times 3$

The number should be of the form $a^1b^1c^2$ where a, b, c are prime numbers.

For number to be least; $c = 2$, $b = 3$, $a = 5$

Least number $= 2^2\, 3^1\, 5^1$

Sum of all divisors

$= (2^0 + 2^1 + 2^2)\,(3^0 + 3^1)\,(5^0 + 5^1) = (7)\,(4)\,(6) = 168$

4. (b) : $2^p + 3^q + 5^r = 2^p + (4-1)^q + (4+1)^r$

$= 2^p + 4\lambda_1 + (-1)^q + 4\lambda_2 + 1^r \quad (\lambda_1, \lambda_2$ are integers)

If $p = 1$, q should be even and r can be any number.

On the other hand if $p \neq 1$, q should be odd and r can be any number.

Total number of ordered triplets

$= 5 \times 10 + 9 \times 5 \times 10 = 500.$

5. (c): Let side $BC = a$ and A be opposite angle

$$R = \frac{a}{2\sin A} \Rightarrow a = 2R\sin A$$

$$\frac{da}{dt} = 2R\cos A\frac{dA}{dt}$$

$$R\frac{dA}{dt} = 2R\cos A\frac{dA}{dt} \qquad \left[\because \frac{da}{dt} = \frac{R\,dA}{dt}\right]$$

$$\Rightarrow \cos A = \frac{1}{2} \Rightarrow A = \frac{\pi}{3}.$$

6. (a) : $\lim\limits_{n\to\infty}\left(n\sqrt[3]{\frac{1}{n}-1+n}\right) = \lim\limits_{n\to\infty}\left[\left(\frac{1}{n}-1\right)^{1/3} + 1^{1/3}\right]$

$$\lim_{n\to\infty} n\left[\frac{\left(\frac{1}{n}-1\right)+1}{\left(\frac{1}{n}-1\right)^{2/3} - \left(\frac{1}{n}-1\right)^{1/3}+1}\right] \left\{\because a+b = \frac{a^3+b^3}{a^2-ab+b^2}\right\}$$

$$\lim_{n\to\infty}\left[\frac{1}{\left(\frac{1}{n}-1\right)^{2/3} - \left(\frac{1}{n}-1\right)^{1/3}+1}\right] = \frac{1}{3}.$$

7. (d) : Here $2a + 2b = x$ *i.e.*, $a + b = \frac{x}{2}$

Area (A) of rectangle $PQRS = PQ \times QR$

$= (PA + AQ)\cdot(QB + BR)$

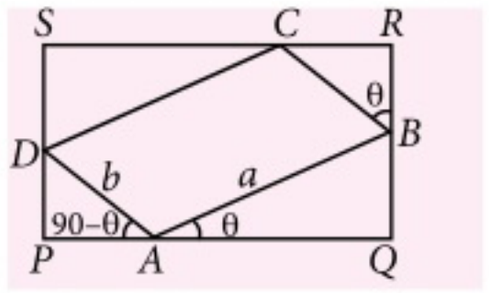
$= (b\sin\theta + a\cos\theta)(a\sin\theta + b\cos\theta)$

$$= ab + \frac{(a^2+b^2)\sin 2\theta}{2}$$

$$A \le ab + \frac{a^2+b^2}{2} \text{ or } A \le \frac{(a+b)^2}{2}$$

$$A_{max} = \frac{(a+b)^2}{2} = 32$$

or, $\frac{x^2}{8} = 32 \quad \therefore x = 16$



8. (c):

$$\frac{e^A}{A} + \frac{e^B}{B} + \frac{e^C}{C} \ge 3\left(\frac{e^{A+B+C}}{ABC}\right)^{1/3}$$

$A + B + C > 3(ABC)^{1/3}$

$$\left(\frac{1}{ABC}\right)^{1/3} \ge \frac{3}{\pi}$$

$$\Rightarrow \left(\frac{e^\pi}{ABC}\right)^{1/3} \ge \frac{3}{\pi}e^{\pi/3} \Rightarrow 3\left(\frac{e^\pi}{ABC}\right) \ge \frac{9}{\pi}e^{\pi/3}$$

$$\Rightarrow \frac{e^A}{A} + \frac{e^B}{B} + \frac{e^C}{C} \ge \frac{9}{\pi}e^{\pi/3}.$$

9. (a) : Total number of ways in which 6 persons can have their birth days

$= 12 \times 12 \times 12 \times 12 \times 12 \times 12 = 12^6$

Out of 12 months, 2 months can be chosen in ${}^{12}C_2$ ways.

Now birth days of six persons can fall in these two months in 2^6 ways. Out of these 2^6 ways, there are two ways when all six birth days fall in one month.

So, there are $(2^6 - 2)$ ways in which six birth days fall in chosen 2 months.

$$\text{Required probability} = \frac{{}^{12}C_2(2^6-2)}{12^6} = \frac{34!}{12^5}$$

10. (a) : $\sin^{-1}\left(\frac{1+x^2}{2x}\right)$ is defined for $\left|\frac{1+x^2}{2x}\right| \le 1$

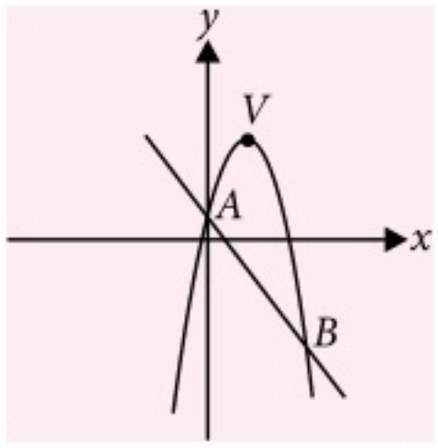
$\Rightarrow x = \pm 1$

Out of these two values of x, only $x = 1$ satisfies the given equation.

OLYMPIAD CORNER

CHALLENGING PROBLEMS FOR RMO, InMO, IMO & OTHER CONTESTS

1. In the diagram, V is the vertex of the parabola with equation $y = -x^2 + 4x + 1$. Also, A and B are the points of intersection of the parabola and the line with equation $y = -x + 1$. Determine the value of $AV^2 + BV^2 - AB^2$.



2. A multiplicative partition of a positive integer $n \geq 2$ is a way of writing n as a product of one or more integers, each greater than 1. Note that we consider a positive integer to be a multiplicative partition of itself. Also, the order of the factors in partition does not matter; for example, $2 \times 3 \times 5$ and $2 \times 5 \times 3$ are considered to be the same partition of 30. For each positive integer $n \geq 2$, define $P(n)$ to be the number of multiplicative partitions of n. We also define $P(1) = 1$. Note that $P(40) = 7$, since the multiplicative partitions of 40 are 40, 2×20, 4×10, 5×8, $2 \times 2 \times 10$, $2 \times 4 \times 5$ and $2 \times 2 \times 2 \times 5$.

(a) Determine the value of $P(64)$.

(b) Determine the value of $P(1000)$.

(c) Determine, with proof, a sequence of integers $a_0, a_1, a_2, a_3, \ldots$ with the property that
$P(4 \times 5^m) = a_0P(2^m) + a_1P(2^{m-1}) + a_2P(2^{m-2}) + \ldots + a_{m-1}P(2^1) + a_mP(2^0)$ for every positive integer m.

3. The line L_1 has equation $y = -\frac{4}{3}x$ and passes through the origin, O. The line L_2 has equation $y = -\frac{1}{2}x + 5$ and crosses the x-axis at P. Lines L_1 and L_2 intersect at Q.

(a) What are the coordinates of points P and Q?

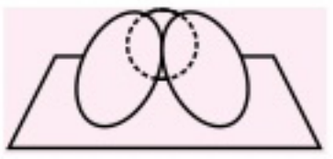
(b) Find the area of ΔOPQ.

(c) Point R is on the positive x-axis so that the area of ΔOQR is three times the area of ΔOPQ. Determine the coordinates of R.

(d) Point S has coordinates $(18, t)$, with $t > 0$. If the area of ΔOQS is three times the area of ΔOPQ, determine the value of t.

4. At Matilda's birthday party, the ratio of people who ate ice cream to people who ate cake was 3 : 2. People who ate both ice cream and cake were included in both categories. If 120 people were at the party, what is the maximum number of people who could have eaten both ice cream and cake?

5. In the diagram, three circles of radius 10 are tangent to each other and to a plane in three-dimensional space. Each of the circles is inclined at 45° to the plane. There are three points where the circles touch each other. These three points lie on a circle parallel to the plane. The radius of this circle is closest to



SOLUTIONS

1. First, we find the coordinates of V.
To do this, we use the given equation for the parabola and complete the square :
$y = -x^2 + 4x + 1 = -(x^2 - 4x - 1)$
$= -(x^2 - 4x + 2^2 - 2^2 - 1) = -((x - 2)^2 - 5)$
$= -(x - 2)^2 + 5$
Therefore, the coordinates of the vertex V are $(2, 5)$.

Next, we find the coordinates of A and B.
Note that A and B are the points of intersection of the line with equation $y = -x + 1$ and the parabola with equation $y = -x^2 + 4x + 1$.
We equate y-values to obtain
$-x + 1 = -x^2 + 4x + 1$ or $x^2 - 5x = 0$
or $x(x - 5) = 0$.
Therefore, $x = 0$ or $x = 5$.
If $x = 0$, then $y = -x + 1 = 1$, and so A (which is on the y-axis) has coordinates $(0, 1)$.
If $x = 5$, then $y = -x + 1 = -4$, and so B has coordinates $(5, -4)$.
We now have the points $V(2, 5)$, $A(0, 1)$, $B(5, -4)$.
This gives

$$AV^2 = (0 - 2)^2 + (1 - 5)^2 = 20$$
$$BV^2 = (5 - 2)^2 + (-4 - 5)^2 = 90$$
$$AB^2 = (0 - 5)^2 + (1 - (-4))^2 = 50$$

and so $AV^2 + BV^2 - AB^2 = 20 + 90 - 50 = 60$

2. In each part, we use "partition" to mean "multiplicative partition". We also call the numbers being multiplied together in a given partition the "parts" of the partition.

(a) We determine the multiplicative partitions of 64 by considering the number of parts in the various partitions. Note that 64 is a power of 2 so any divisor of 64 is also a power of 2. In each partition, since the order of parts is not important, we list the parts in increasing order to make it easier to systematically find all of these.
One part : There is one possibility : 64.
Two parts : There are three possibilities: $64 = 2 \times 32 = 4 \times 16 = 8 \times 8$.
Three parts : We start with the smallest possible first and second parts. We keep the first part fixed while adjusting the second and third parts. We then increase the first part and repeat.
We get: $64 = 2 \times 2 \times 16 = 2 \times 4 \times 8$
$= 4 \times 4 \times 4$.
Four parts : A partition of 64 with four parts must include at least two 2s, since if it didn't, it would include at least three parts that are at least 4, and so would be too large. With two 2s, the remaining two parts have a product of 16.
We get: $64 = 2 \times 2 \times 2 \times 8 = 2 \times 2 \times 4 \times 4$.
Five parts: a partition of 64 with five parts must include at least three 2s, since if it didn't, it would include at least three parts that are at least 4, and so would be too large. With three 2s, the remaining two parts have a product of 8.
We get: $64 = 2 \times 2 \times 2 \times 2 \times 4$.
Six parts: Since $64 = 2^6$, there is only one possibility: $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$.
Therefore, $P(64) = 1 + 3 + 3 + 2 + 1 + 1 = 11$.

(b) First, we note that
$1000 = 10^3 = (2 \cdot 5)^3 = 2^3 5^3$.
We calculate the value of $P(p^3q^3)$ for two distinct prime numbers p and q. It will turn out that this value does not depend on p and q. This value will be the value of $P(1000)$, since 1000 has this form of prime factorization.
Let $n = p^3q^3$ for distinct prime numbers p and q.
The integer n has three prime factors equal to p.
In a given partition, these can be all together in one part (as p^3), can be split between two different parts (as p and p^2), or can be split between three different parts (as p, p and p).
There are no other ways to divide up three divisors of p.
Similarly, n has three prime factors equal to q which can be divided in similar ways.
We determine $P(p^3q^3)$ by considering the possible combination of the number of parts divisible by p and the number of parts divisible by q and counting partitions in each case. In other words, we complete the following table:

Number of parts divisible by q \ Number of parts divisible by p	1	2	3
1			
2			
3			

We not that the table is symmetric, since the factors of p and q are interchangeable. We proceed to consider cases, considering only those on the top left to bottom right diagonal and those below this diagonal in the table.
Case 1: One part divisible by p, one part divisible by q.
The partition must be p^3q^3 (n itself) or $p^3 \times q^3$.
There are two partitions in this case.

Case 2: One part divisible by p, two parts divisible by q

The three factors of p occur together as p^3. The three factors of q occur as q and q^2.

The p^3 can occur in one of the parts divisible by q or not.

This give partitions $p^3 \times q \times q^2$ and $p^3q \times q^2$ and $q \times p^3q^2$.

There are three partitions in this case. Similarly, there are three partitions with one part divisible by q and two parts divisible by p.

Case 3: One part divisible by p, three parts divisible by q

The three factors of p occur together as p^3. The three factors of q occur as q, q and q.

The p^3 can occur in one of the parts divisible by q or not.

This gives partitions $p^3 \times q \times q \times q$ and $p^3q \times q \times q$.

(Note that the three divisors of q are interchangeable so p^3 only needs to be placed with one of them.)

There are two partitions in this case. Similarly, there are two partitions with one part divisible by q and three parts divisible by p.

Case 4: Two parts divisible by p, two parts divisible by q

The three factors of p occur as p and p^2. The three factors of q occur as q and q^2.

Each of p and p^2 can occur in one of the parts divisible by q or not.

It no part is a multiple of both p and q, we have one partition: $p \times p^2 \times q \times q^2$.

If one part is a multiple of both p and q, there are two choices for which power of p to include in this part and two choices for which power of q to include (there is no choice for the remaining parts). Thus, there are $2 \times 2 = 4$ such partitions:

$p^2q^2 \times p \times q$ $\quad$ $pq^2 \times p^2 \times q$

$p^2q \times p \times q^2$ $\quad$ $pq \times p^2 \times q^2$

If two parts are a multiple of both p and q, there are two ways to choose the power of p in the part containing just q, so there are two such partitions: $pq \times p^2q^2$ and $p^2q \times pq^2$.

There are seven partitions in this case.

Case 5: Two parts divisible by p, three parts divisible by q

The three factors of p occur as p and p^2. The three factors of q occurs as q, q and q.

Each of p and p^2 can occur in one of the parts divisible by q or not.

If no part is a multiple of both p and q, we have one partition: $p \times p^2 \times q \times q \times q$.

If one part is a multiple of both p and q, there are two choices for which power of p to include in this part (since all powers of q are identical).

Thus, there are 2 such partitions:

$p^2q \times p \times q \times q$ and $pq \times p^2 \times q \times q$.

It two parts are a multiple of both p and q, there is one partition, since all of the powers of q are identical: $pq \times p^2q \times q$.

There are four partitions in this case. Similarly, there are four partitions with two parts divisible by q and three parts divisible by p.

Case 6: Three parts divisible by p, three parts divisible by q

The three factors of p as p, p and p. The three factors of q appear as q, q and q.

Here, the number of parts in the partition that are multiples of both p and q can be 0, 1, 2 or 3. Since all of the powers of p and q are identical, the partitions are completely determined by this and are

$p \times p \times p \times q \times q \times q$ $\quad$ $p \times p \times pq \times q \times q$

$p \times pq \times pq \times q$ $\quad$ $pq \times pq \times pq$

There are four partitions in this case.

Finally, we complete the table:

Number of parts divisible by q \ Number of parts divisible by p	1	2	3
1	2	3	4
2	3	7	4
3	2	4	4

MtG

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Adding the entries in the table, we obtain $P(p^3q^3) = 31$.

Thus, $P(1000) = 31$.

(c) As in (b), the value of $P(n)$ depends only on the structure of the prime factorization of n, not on the actual primes in the factorization.

Therefore, $P(4 \times 5^m) = P(2^2 \times 5^m) = P(p^2q^m)$ for any distinct primes p and q.

Therefore, $P(4 \times 5^m) = P(p^2q^m)$

$= P(5^2 \times 2^m) = P(25 \times 2^m)$.

We count the number of multiplicative partitions of $N = 5^2 \times 2^m$ by considering the placement of the $2s$ and $5s$ among the parts of the partitions.

Since N has only two factors of 5, these can occur in the same part, or in different parts.

Note that every factor of N is a product is of the form 5^j2^k for some integers j and k with $0 \le j \le 2$ and $0 \le k \le m$.

We first count the number of partitions where the two factors of 5 occur in the same part.

Consider one such partition.

In this partition, the part containing the two $5s$ will be of the form 5^22^k for some integer k with $0 \le k \le m$.

Thus, this partition will be of the form $5^22^k \times P$, where P is a partition of 2^{m-k} (the remaining factors in N).

Since the order of parts does not matter, there are $P(2^{m-k})$ such partitions P, and so this number of partitions of N of this form.

Since k ranges from 0 to m, then the number of partitions where the two $5s$ occur in the same part equals

$P(2^m) + P(2^{m-1}) + \ldots + P(2^1) + P(2^0)$

Next, we count the number of partitions where the two factors of 5 occur in different parts.

Consider one such partition.

In this partition, the parts containing the two $5s$ will be of the form 5×2^a and 5×2^b for some integers a and b with $0 \le a$, $m \le b$ and $a + n \le b$.

Since the order of the parts within a partition does not matter, we can restrict a and b further by requiring that $0 \le a \le b \le m$ and $a + b \le m$ to avoid double-counting partitions.

Thus, this partition will be of the form $(5 \times 2^a) \times (5 \times 2^b) \times P$, where P is a partition of 2^{m-a-b} (the remaining factors in N).

Since the order of parts does not matter, there are $P(2^{m-a-b})$ such partitions P, and so this number of partitions of N of this form.

To determine the total number of partitions in this case, we need to add up $P(2^{m-a-b})$ over all possible pairs (a, b) satisfying $0 \le a \le b \le m$ and $a + b \le m$.

To do this, we focus on the possible values of $s = a + b$ and count the number of pairs (a, b) that give this sum.

If $s = a + b = 0$, there is one pair (a, b), namely $(a, b) = (0, 0)$.

If $s = a + b = 1$, there is one pair (a, b), namely $(a, b) = (0, 1)$.

If $s = a + b = 2$, there two pairs (a, b), namely $(a, b) = (0, 2), (1, 1)$.

In general, if s is even, then $\frac{1}{2}s$ is an integer and so there are $(\frac{1}{2}s + 1)$ pairs (a, b), namely

$(0, s), (1, s - 1), (2, s - 2), \ldots,$

$(\frac{1}{2}s - 1, \frac{1}{2}s + 1), (\frac{1}{2}s, \frac{1}{2}s)$

Any larger value of a would give a value of b smaller than a.

In general, if s is odd, then $\frac{1}{2}s - \frac{1}{2}$ is an integer and so there are $(\frac{1}{2}s - \frac{1}{2}) + 1 = (\frac{1}{2}s + \frac{1}{2})$ pairs (a, b), namely

$(0, s), (1, s - 1), (2, s - 2), \ldots,$

$(\frac{1}{2}s - \frac{3}{2}, \frac{1}{2}s + \frac{3}{2}), (\frac{1}{2}s - \frac{1}{2}, \frac{1}{2}s + \frac{1}{2})$

Any larger value of a would give a value of b smaller than a.

To summarize, if $s = a + b$ is even, there are $(\frac{1}{2}s + 1)$ pairs (a, b) and if $s = a + b$ is odd, there are $(\frac{1}{2}s + \frac{1}{2})$ pairs (a, b).

Thus, as s increases from 0, the number of pairs (a, b) gives the sequence 1, 1, 2, 2, 3, 3,.... The number in this sequence corresponding to the value of $a + b$ gives the number of times

that $P(2^{m-a-b})$ should be included in the count of the total number of partitions in this case.

In other words, if $a + b = 0$, there are $1 \times P(2^m)$ partitions, if $a + b = 1$, there are $1 \times P(2^{m-1})$ partitions, if $a + b = 2$, there are $2 \times P(2^{m-2})$ partitions, etc.

We can rewrite this more compactly to say that for a given s, the number of pairs (a, b) is $\left\lfloor \frac{s+2}{2} \right\rfloor$ (where $\lfloor x \rfloor$ is the greatest integer less than or equal to x) and so the number of partitions is $\left\lfloor \frac{s+2}{2} \right\rfloor \times P(2^{m-s})$.

Therefore, the total number of partitions of N in this case is

$$1 \times P(2^m) + 1 \times P(2^{m-1}) + 2 \times P(2^{m-2}) + 2 \times P(2^{m-3}) + \ldots + \left\lfloor \frac{s+2}{2} \right\rfloor \times P(2^{m-s}) + \ldots + \left\lfloor \frac{m}{2} \right\rfloor \times P(2^1) + \left\lfloor \frac{m+2}{2} \right\rfloor \times P(2^0)$$

Combining the two cases and adding the corresponding expressions for the number of partitions, we obtain the total number of partitions is

$$2 \times P(2^m) + 2 \times P(2^{m-1}) + 3 \times P(2^{m-2}) + 3 \times P(2^{m-3}) + \ldots + \left(1+\left\lfloor \frac{s+2}{2} \right\rfloor\right) \times P(2^{m-s}) + \ldots + \left(1+\left\lfloor \frac{m+1}{2} \right\rfloor\right) \times P(2^1) + \left(1+\left\lfloor \frac{m+2}{2} \right\rfloor\right) \times P(2^0)$$

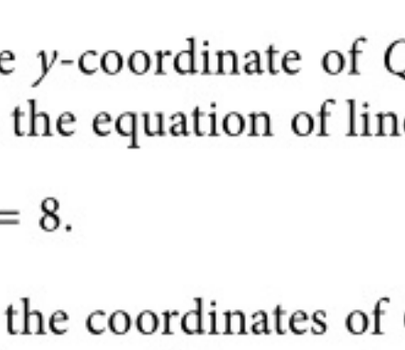
and so the desired sequence is

$a_0 = 2$
$a_1 = 2$
$a_2 = 3$
$a_3 = 3$
$\vdots$
$a_s = 1 + \left\lfloor \frac{s+2}{2} \right\rfloor$
$\vdots$

3.(a) To find the coordinates of P, the point where line L_2 crosses the x-axis, we set $y = 0$ and solve for x.

Thus, we solve $0 = -\frac{1}{2}x + 5$ or $\frac{1}{2}x = 5$ to get $x = 10$.

Therefore, the coordinates of P are $(10, 0)$.



To find the coordinates of Q, the point of intersection of lines L_1 and L_2, we equate values of y and solve for x.

Thus, we solve $-\frac{4}{3}x = -\frac{1}{2}x + 5$. We multiply both sides by 6 to obtain $6(-\frac{4}{3}x) = 6(-\frac{1}{2}x + 5)$

or $-8x = -3x + 30$, which gives $-5x = 30$

or $x = 6$.

To find the y-coordinate of Q, we substitute $x = -6$ into the equation of line L_1 to obtain $y = -\frac{4}{3}(-6) = 8$.

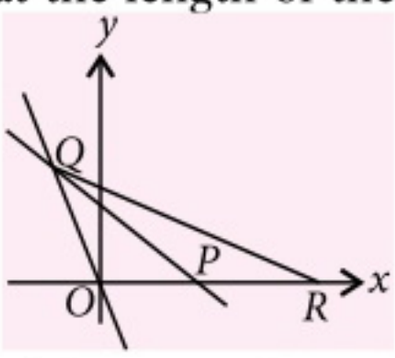
Therefore, the coordinates of Q are $(-6, 8)$.

(b) The coordinates of the vertices of ΔOPQ are $O(0, 0)$, $P(10, 0)$ and $Q(-6, 8)$.

We can view this triangle as having base OP along the x-axis. OP has length 10. The height of this triangle is then the distance from Q to the x-axis, or 8.

Therefore, the area ΔOPQ is $\frac{1}{2}(10)(8) = 40$.

(c) Since the area of ΔOPQ is 40 and the area of ΔOQR is three times the area of, ΔOQR then the area of ΔOQR is $3 \times 40 = 120$. Since R is to be on the positive x-axis, then we can view ΔOQR as having base OR along the x-axis. Suppose that the length of the base is b.



The height of this triangle is again the distance from Q to the x-axis, or 8.

Thus, we want $\frac{1}{2}b(8) = 120$ or $4b = 120$, and so $b = 30$.

Since O has coordinates $(0, 0)$ and R is on the positive x-axis, then R must have coordinates $(30, 0)$.

(We could also have noted that ΔOQR and ΔOPQ can be viewed as having the same height, so for the areas to be in the ratio 3 : 1, then the ratio of the lengths of the bases must also be 3 : 1.)

(d) Since the area of ΔOQS is to be three times the area of ΔOPQ, then its area is to be 120, which is equal to the area of ΔOQR from (c). Here are two methods to find the value of t.

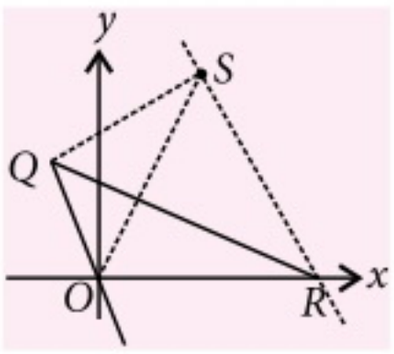
Method 1: Constant heights

Consider ΔOQS and ΔOQR.

These triangles are to have the same area, and can be viewed as having a common base OQ. Therefore, the two triangles must have the same height.

This height will be the perpendicular distance from $R(30, 0)$ to the line $y = -\frac{4}{3}x$, along which OQ lies.

We find the coordinates of S (knowing that it must have an x-coordinate of 18) by sliding a point from R along a line parallel to the base OQ (so along a line with slope $-\frac{4}{3}$). This process will maintain the height.

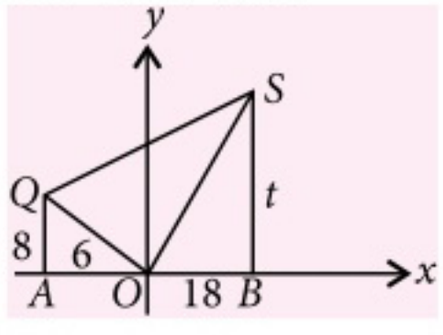


To get from R to S, we slide $30 - 18 = 12$ units to the left. Since the slope of the line is $-\frac{4}{3}$, then we must go up by $\frac{4}{3}(12) = 16$ units.

Therefore, S has coordinates (18, 16); that is, $t = 16$.

Method 2: Using a trapezoid

Join point O, Q and S to each other, and draw perpendiculars from Q to $A(-6, 0)$ and from S to $B(18, 0)$ on the x-axis.



Then the area of trapezoid $AQSB$ will equal the sum of the areas of ΔAQO, ΔOQS and ΔOSB.

ΔAQO has base 6 and height 8 (since the coordinates of Q are $(-6, 8)$) and so ΔAQO has area $\frac{1}{2}(6)(8) = 24$.

ΔOQS is to have area 120.

ΔOSB has base 18 and height t (since the coordinates of S are $(18, t)$) and so ΔOSB has area $\frac{1}{2}(18)t = 9t$.

Thus, trapezoid $AQSB$ has area $24 + 120 + 9t$.

Viewed another way, trapezoid $AQSB$ has parallel sides AQ and BS of lengths 8 and t, and height AB of length $6 + 18 = 24$, and so has area $\frac{1}{2}(8 + t)(24) = 96 + 12t$.

Equating these areas, we obtain $96 + 12t = 24 + 120 + 9t$ and so $3t = 48$ or $t = 16$.

4. Since we want the maximum number of people, we start with the largest of the given choices and see if we can make it work.

Is it possible that 80 people could have eaten both ice cream and cake?

If so, there would be at least 80 people who ate cake and at least $\frac{3}{2}(80) = 120$ people who ate ice cream. (Note that there can be overlap between these two sets of people.)

Is this possible?

It is if we say that exactly 120 people ate ice cream and exactly 80 people ate cake, with all of these 80 eating ice cream as well.

So an overlap of 80 is possible, which means that the answer is (D).

5. Let the three points at which the circles are tangent to the plane be A, B and C.

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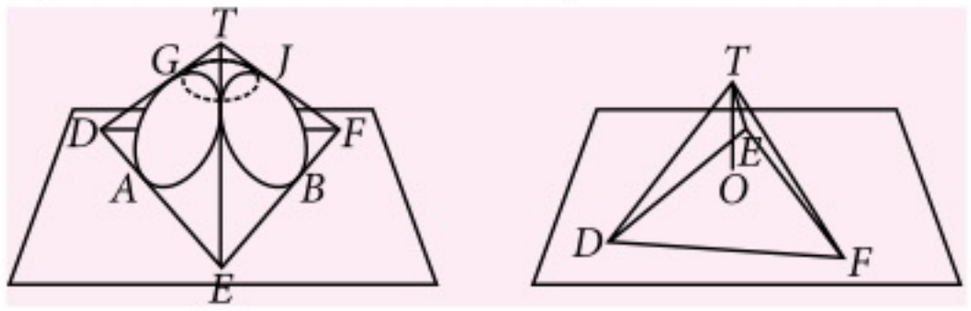
Each of the three circles is contained in a plane. This plane will intersect the original plane along a line that passes through A, B or C, and is tangent to the circle at one of these points. Suppose that these three lines intersect at points D, E and F, with A on DE, B on EF, and C on FD.

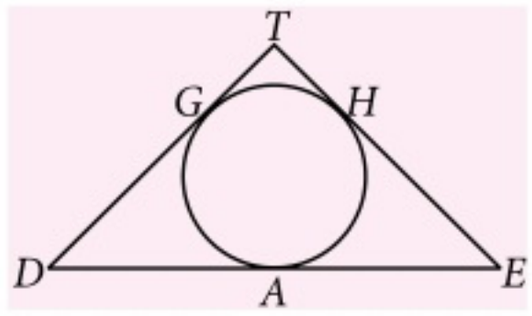
By symmetry, $DE = EF = FD$. Also, A, B and C are the midpoints of these three segments. (More formally, the configuration is not changed by rotating it through 120° or by reflecting it horizontally through the three vertical planes that pass through T (defined below) and the vertices of ΔDEF, so these facts are true.)

Thus, ΔDEF is equilateral. Let O be its incentre. That is, O is the point of intersection of the three angle bisectors of ΔDEF. Since ΔDEF is equilateral, then its angle bisectors, medians and altitudes are all the same.

Let the points where the three circles touch be G, H and J, with G the point of contact between the circles containing C and A, H the point of contact between the circles containing A and B, and J the point of contact between the circles containing B and C. Note that $GH = HJ = JG$ by symmetry. We want to determine the radius of the circle that passes through G, H and J. Draw the lines through D, E and F passing through G, H and J, respectively. These lines will be tangent to the circles at points G, H and J, and will meet at a common point, which we call T. T is directly above point O, the centre of ΔDEF. (Each of these facts is again true by the symmetry of rotation and reflection.) Now, $TDEF$ is a tetrahedron with its base (ΔDEF) equilateral, and three congruent side faces.



Consider the side face ΔDET. The circle (of radius 10) containing A is tangent to the three sides of ΔDET at A, G and H.

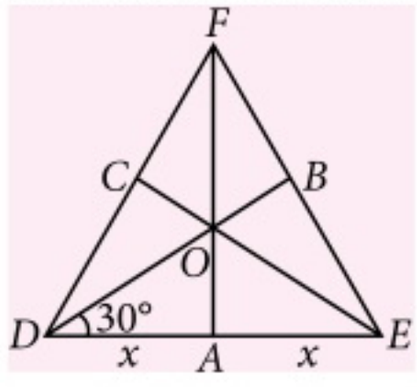


Also, ΔDET is inclined at 45° to the horizontal. Since A is the midpoint of DE, then OA is perpendicular to DE, as is TA.

Since ΔDET is inclined at 45° to the horizontal, then $\angle TAO = 45°$.

Note that ΔTAO is right-angled at O (because T is directly above O), so the fact that $\angle TAO = 45°$ tells us that ΔTAO is a 45°-45°-90° triangle, and so $TA = \sqrt{2}\, AO$.

Next, we look at the dimensions of ΔDEF. Suppose that $DE = 2x$. Thus, $DA = AE = x$ and ΔDEF is equilateral with side length $2x$.



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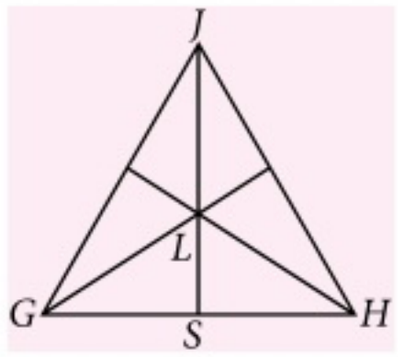
Now DO is the angle bisector of $\angle EDF$, so $\angle ODA = 30°$, and OA is perpendicular to DE, so ΔDOA is a 30°-60°-90° triangle. Thus,

$$OA = \frac{1}{\sqrt{3}}DA = \frac{1}{\sqrt{3}}x, \text{ and } OD = \frac{2}{\sqrt{3}}DA = \frac{2}{\sqrt{3}}x.$$

Since $TA = \sqrt{2}\,AO$, then

$$TA = \frac{\sqrt{2}}{\sqrt{3}}x.$$



In ΔTAD, the Pythagorean Theorem gives

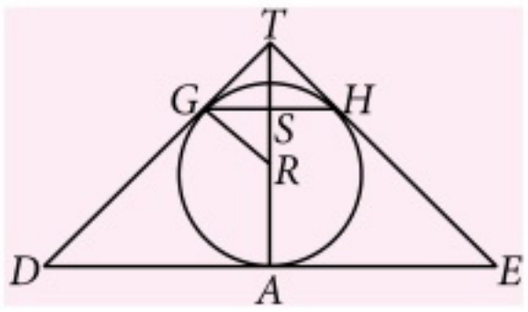
$$TD = \sqrt{TA^2 + DA^2} = \sqrt{x^2 + \frac{2}{3}x^2} = \frac{\sqrt{5}}{\sqrt{3}}x,$$

since $x > 0$.

So, to summarize so far, ΔDET has $DE = 2x$, $DA = AE = x$, $TD = \frac{\sqrt{5}}{\sqrt{3}}x$ and $TA = \frac{\sqrt{2}}{\sqrt{3}}x$.

Let the centre of the circle contained in ΔDET be R (which will lie on TA by symmetry) and join R to G. Since RG is a radius of the circle, then $RG = 10$ and RG is perpendicular to DT.

Also, join GH and let the point of intersection of GH with TA be S. By symmetry, GH is perpendicular to TA.



Now ΔTSG, ΔTGR and ΔTAD are all similar, since each is right-angled and shares a common angle at T.

We want to determine the length of SG.

By similar triangles, $\frac{SG}{TG} = \frac{AD}{TD} = \frac{x}{\frac{\sqrt{5}}{\sqrt{3}}x}$,

so $SG = \frac{\sqrt{3}}{\sqrt{5}}TG$.

Also by similar triangles, $\frac{TG}{GR} = \frac{TA}{AD} = \frac{\frac{\sqrt{2}}{\sqrt{3}}x}{x}$,

so $TG = \frac{\sqrt{2}}{\sqrt{3}}GR = 10\frac{\sqrt{2}}{\sqrt{3}}$.

Thus, $SG = \frac{\sqrt{3}}{\sqrt{5}}\left(10\frac{\sqrt{2}}{\sqrt{3}}\right) = 10\frac{\sqrt{2}}{\sqrt{5}} = 2\sqrt{10}$.

Finally, consider ΔGHJ. This triangle is equilateral. S is the midpoint of GH and $SG = 2\sqrt{10}$.

Let L be the incentre of this triangle.

Note that L is also the centre of the circle that passes through G, H and J because $LG = LJ = LH$, so LG is the radius of this circle.

Since above we saw that $OD = \frac{2}{\sqrt{3}}AD$, then $LG = \frac{2}{\sqrt{3}}SG$, since the configuration is the same in ΔDEF and ΔGHJ. Thus, $LG = \frac{4\sqrt{10}}{\sqrt{3}} = \frac{4}{3}\sqrt{30}$, and this is the radius of the circle.

Since $\frac{4}{3}\sqrt{30} \approx 7.303$, the answer is closest to 7.3, of the given choices.

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concept BOOSTERS

Class XI

Straight Lines

* ALOK KUMAR, B.Tech, IIT Kanpur

This column is aimed at Class XI students so that they can prepare for competitive exams such as JEE Main/Advanced, etc. and be also in command of what is being covered in their school as part of NCERT syllabus. The problems here are a happy blend of the straight and the twisted, the simple and the difficult and the easy and the challenging.

DISTANCE FORMULA

The distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

SECTION FORMULA

If $P(x, y)$ divides the line joining $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$, then

$$x = \frac{m x_2 + n x_1}{m + n}; \quad y = \frac{m y_2 + n y_1}{m + n}$$

- If $\frac{m}{n}$ is positive, the division is internal. If $\frac{m}{n}$ is negative, the division is external.
- If P divides AB internally in the ratio $m : n$ and Q divides AB externally in the ratio $m : n$, then P and Q are said to be harmonic conjugate of each other w.r.t. AB.

$$\frac{2}{AB} = \frac{1}{AP} + \frac{1}{AQ}$$ *i.e.*, AP, AB and AQ are in H.P.

CENTROID AND INCENTRE

If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of triangle ABC, whose sides BC, CA, AB are of lengths a, b, c respectively, then the coordinates of the centroid are : $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

and the coordinates of the incentre are : $\left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c}\right)$

Note :

Incentre divides the angle bisectors in the ratio $(b + c) : a$; $(c + a) : b$ and $(a + b) : c$.

- Orthocentre , Centroid and Circumcentre are always collinear and centroid divides the line joining orthocentre and circumcentre in the ratio 2 : 1.

SLOPE FORMULA

If θ is the angle at which a straight line is inclined with the positive direction of the x-axis, in anti-clockwise sense then the slope of the line, denoted by m, is defined by $m = \tan\theta$. If θ is 90°, m does not exist, but the line is parallel to the y-axis.

If θ = 0, then $m = 0$ and the line is parallel to the x-axis.

If $A(x_1, y_1)$ and $B(x_2, y_2)$, $x_1 \neq x_2$, are points on a straight line, then the slope m of the line is given by

$$m = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

CONDITION OF COLLINEARITY OF THREE POINTS - (SLOPE FORM)

Points $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are collinear if $\left(\frac{y_2 - y_1}{x_2 - x_1}\right) = \left(\frac{y_3 - y_1}{x_3 - x_1}\right)$.

EQUATION OF A STRAIGHT LINE IN VARIOUS FORMS

- **Slope-Intercept Form :** $y = mx + c$ is the

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equation of a straight line whose slope is m and which makes an intercept c on the y-axis.

- **Slope One Point Form:** $y - y_1 = m(x - x_1)$ is the equation of a straight line whose slope is m and which passes through the point (x_1, y_1).
- **Parametric Form :** The equation of the line in parametric form is given by $\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$ (say). Where 'r' is the distance of any point (x, y) on the line from the fixed point (x_1, y_1) on the line. r is positive if the point (x, y) is on the right of (x_1, y_1) and negative if (x, y) lies on the left of (x_1, y_1).
- **Two Point Form :** $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ is the equation of a straight line which passes through the points (x_1, y_1) and (x_2, y_2) .
- **Intercept Form :** $\frac{x}{a} + \frac{y}{b} = 1$ is the equation of a straight line which makes intercepts a and b on OX and OY respectively .
- **Perpendicular Form :** $x\cos\alpha + y\sin\alpha = p$ is the equation of the straight line where the length of the perpendicular from the origin O on the line is p and this perpendicular makes angle α with positive side of x-axis.
- **General Form :** $ax + by + c = 0$ is the equation of a straight line in the general form.

POSITION OF THE POINT (X_1, Y_1) RELATIVE TO THE LINE $AX + BY + C = 0$

If $ax_1 + by_1 + c$ is of the same sign as c, then the point (x_1, y_1) lie on the origin side of $ax + by + c = 0$. But if the sign of $ax_1 + by_1 + c$ is opposite to that of c, the point (x_1, y_1) will lie on the non-origin side of $ax + by + c = 0$.

THE RATIO m : n IN WHICH A GIVEN LINE DIVIDES THE LINE SEGMENT JOINING TWO POINTS

Let the given line $ax + by + c = 0$ divide the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$, then $\frac{m}{n} = -\frac{a x_1 + b y_1 + c}{a x_2 + b y_2 + c}$.

If A and B are on the same side of the given line then $\frac{m}{n}$ is negative but if A and B are on opposite sides of the given line, then $\frac{m}{n}$ is positive.

LENGTH OF PERPENDICULAR FROM A POINT ON A LINE

The length of perpendicular from $P(x_1, y_1)$ on $ax + by + c = 0$ is $\left|\frac{a x_1 + b y_1 + c}{\sqrt{a^2 + b^2}}\right|$.

ANGLE BETWEEN TWO STRAIGHT LINES IN TERMS OF THEIR SLOPES

If m_1 and m_2 are the slopes of two intersecting straight lines $(m_1 m_2 \neq -1)$ and θ is the acute angle between them, then $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$.

Let m_1, m_2, m_3 are the slopes of three lines $L_1 = 0$; $L_2 = 0$; $L_3 = 0$ where $m_1 > m_2 > m_3$ then the interior angles of the ΔABC found by these lines are given by,

$$\tan A = \frac{m_1 - m_2}{1 + m_1 m_2}; \quad \tan B = \frac{m_2 - m_3}{1 + m_2 m_3}$$

and $\tan C = \frac{m_3 - m_1}{1 + m_3 m_1}$

PARALLEL LINES

- When two straight lines are parallel their slopes are equal. Thus any line parallel to $ax + by + c_1 = 0$ is of the type $ax + by + c_2 = 0$
- The distance between two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $\left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$.
- The area of the parallelogram $= \frac{p_1 p_2}{\sin\theta}$, where p_1 and p_2 are distances between two pairs of opposite sides and θ is the angle between any two adjacent sides.

Note that area of the parallelogram bounded by the lines $y = m_1x + c_1$, $y = m_1x + c_2$ and $y = m_2x + d_1$, $y = m_2x + d_2$ is given by $\left|\frac{(c_1 - c_2)(d_1 - d_2)}{m_1 - m_2}\right|$.

PERPENDICULAR LINES

- When two lines of slopes m_1 and m_2 are at right angles, the product of their slopes is -1, *i.e.* $m_1\ m_2 = -1$. Thus any line perpendicular to $ax + by + c = 0$ is of the form $bx - ay + k = 0$, where k is any parameter.
- Straight lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ are at right angles if and only if $aa' + bb' = 0$.
- Equations of straight lines through (x_1, y_1) making angle α with $y = mx + c$ are
  $(y - y_1) = \tan(\theta - \alpha)(x - x_1)$ and
  $(y - y_1) = \tan(\theta + \alpha)(x - x_1)$, where $\tan\theta = m$.

CONDITION OF CONCURRENCY

Three lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ are concurrent if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

If three constants A, B and C can be found such that $A(a_1x + b_1y + c_1) + B(a_2x + b_2y + c_2) + C(a_3x + b_3y + c_3) \equiv 0$, then the three straight lines are concurrent.

AREA OF A TRIANGLE

If (x_i, y_i), $i = 1, 2, 3$ are the vertices of a triangle, then its area is equal to $\frac{1}{2}\left|\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right|$ provided the vertices are considered in the counter clockwise sense. The above formula will give a $(-)$ ve area if the vertices (x_i, y_i), $i = 1, 2, 3$ are placed in the clockwise sense.

CONDITION OF COLLINEARITY OF THREE POINTS–(AREA FORM)

The points (x_i, y_i), $i = 1, 2, 3$ are collinear if $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$

THE EQUATION OF A FAMILY OF STRAIGHT LINES PASSING THROUGH THE POINTS OF INTERSECTION OF TWO GIVEN LINES

The equation of a family of lines passing through the point of intersection of
$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is given by $(a_1x + b_1y + c_1) + k(a_2x + b_2y + c_2) = 0$, where k is an arbitrary real number.

BISECTORS OF THE ANGLES BETWEEN TWO LINES

- Equation of the bisectors of angles between the lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ $(ab' \neq a'b)$ are : $\frac{ax + by + c}{\sqrt{a^2 + b^2}} = \pm\frac{a'x + b'y + c'}{\sqrt{a'^2 + b'^2}}$

TO DISCRIMINATE BETWEEN THE ACUTE ANGLE(INTERNAL) BISECTOR AND THE OBTUSE (EXTERNAL) ANGLE BISECTOR

If θ be the angle between one of the lines and one of the bisectors, find $\tan\theta$.
If $|\tan\theta| < 1$, then $2\theta < 90°$ so that this bisector is the acute angle bisector .
If $|\tan\theta| > 1$, then we get the bisector to be the obtuse angle bisector.

BISECTOR OF THE ANGLE BETWEEN THE TWO LINES CONTAINING THE ORIGIN

$$\frac{ax + by + c}{\sqrt{a^2 + b^2}} = +\frac{a'x + b'y + c'}{\sqrt{a'^2 + b'^2}};\ c.c' > 0$$

BISECTOR OF THE ANGLE BETWEEN THE TWO LINES DOES NOT CONTAINING THE ORIGIN

$$\frac{ax + by + c}{\sqrt{a^2 + b^2}} = -\frac{a'x + b'y + c'}{\sqrt{a'^2 + b'^2}};\ c.c' > 0$$

SHORTCUT METHOD FOR FINDING ACUTE (INTERNAL) AND OBTUSE (EXTERNAL) ANGLE BISECTORS

The equation of the bisectors are

$$\frac{ax + by + c}{\sqrt{a^2 + b^2}} = \pm\frac{a'x + b'y + c'}{\sqrt{a'^2 + b'^2}}, \text{ then}$$

Conditions	Acute angle Bisector	Obtuse angle Bisector
$a_1a_2 + b_1b_2 > 0$	$-$	$+$
$a_1a_2 + b_1b_2 < 0$	$+$	$-$

Note : (i) Bisectors are perpendiculars to each other.
(ii) '+' sign gives the bisector of the angle containing origin.

(iii) If $a_1a_2 + b_1b_2 > 0$, then the origin lies in obtuse angle and if $a_1a_2 + b_1b_2 < 0$, then the origin lies in acute angle.

A PAIR OF STRAIGHT LINES THROUGH ORIGIN

- A homogeneous equation of degree two of the type $ax^2 + 2hxy + by^2 = 0$ always represents a pair of straight lines passing through the origin and if
  - $h^2 > ab \Rightarrow$ lines are real and distinct.
  - $h^2 = ab \Rightarrow$ lines are coincident.
  - $h^2 < ab \Rightarrow$ lines are imaginary with real point of intersection *i.e.* (0, 0)
- If $y = m_1x$ and $y = m_2x$ be the two equations represented by $ax^2 + 2hxy + by^2 = 0$, then;

$$m_1 + m_2 = -\frac{2h}{b} \text{ and } m_1m_2 = \frac{a}{b}$$

- If θ is the acute angle between the pair of straight lines represented by $ax^2 + 2hxy + by^2 = 0$, then,

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{|a + b|}$$

The condition that these lines are:

- At right angles to each other is $a + b = 0$. *i.e.* co–efficient of x^2 + coefficient of $y^2 = 0$.
- Coincident is $h^2 = ab$.

A homogeneous equation of degree n represents n straight lines passing through origin.

GENERAL EQUATION OF SECOND DEGREE REPRESENTING A PAIR OF STRAIGHT LINES

- $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0,$$

$$\text{i.e. if } \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

- The angle θ between the two lines representing by a general equation is the same as that between the two lines represented by its homogeneous part only .
- The joint equation of a pair of straight lines joining origin to the points of intersection of the line given by $lx + my + n = 0$ .... (i) and the 2nd degree curve :

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ ... (ii)

is

$$ax^2 + 2hxy + by^2 + 2gx\left(\frac{lx + my}{-n}\right) + 2fy\left(\frac{lx + my}{-n}\right) + c\left(\frac{lx + my}{-n}\right)^2 = 0 \text{ ... (iii)}$$

is obtained by homogenizing (ii) with the help of (i), by writing (i) in the form :

$$\left(\frac{lx + my}{-n}\right) = 1$$

- The equation to the straight lines bisecting the angle between the straight lines,

$ax^2 + 2hxy + by^2 = 0$ is $\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$.

- The product of the perpendiculars, dropped from (x_1, y_1) to the pair of lines represented by the equation, $ax^2 + 2hxy + by^2 = 0$ is

$$\frac{ax_1^2 + 2hx_1y_1 + by_1^2}{\sqrt{(1-b)^2 + 4h^2}} .$$

- Any second degree curve through the four point of intersection of $f(xy) = 0$ and $xy = 0$ is given by $f(xy) + \lambda xy = 0$ where $f(xy) = 0$ is also a second degree curve.

PROBLEMS

SECTION-I

Single Correct Answer Type

1. Let two points $A(1, 2)$, $B(7, 5)$. The line AB is rotated through 45° in anticlockwise direction about the point of trisection of AB which is nearest to B. The equation of the line in new position is

(a) $2x - y - 6 = 0$ (b) $3x - y - 11 = 0$

(c) $x - 3y - 1 = 0$ (d) $x - y - 1 = 0$

2. If $L_1 = 0$ and $L_2 = 0$ are two intersecting lines such that the reflection of L_1 in L_2 and the reflection of L_2 in L_1 coincide, then the angle between the lines is

(a) 30° (b) 48° (c) 60° (d) 90°

3. Two fixed points A and B are taken on the rectangular coordinate axes such that $OA = a$ and $OB = b$. Two variable points P and Q are taken on the same axes so that $OP + OQ = OA + OB$. The locus of the intersection of AQ and BP is

(a) $x - y = a$ (b) $x + y = a + b$

(c) $x - y = b - a$ (d) $x - y = b$

4. The lines $ax + by + (a\alpha + b) = 0$, $bx + cy + (b\alpha + c) = 0$ and $(a\alpha + b)x + (b\alpha + c)y = 0$ are concurrent if

(a) a, b, c are in AP

(b) $(x - \alpha)$ is a factor of $ax^2 + 2bx + c$

(c) α is a factor of $ax^2 + bx + c = 0$

(d) a, b, c are in HP

5. If $x^2 - 3xy + 2y^2 = 0$ represents two sides of triangle and orthocentre is (2,1), then equation of third side is

(a) $x + 2y = 5$ (b) $2x + y = 1$

(c) $2x + y = 5$ (d) $2x + y = 6$

6. If $2x + 3y = 7$ makes equal angles with pair of lines $9x^2 + 12xy + ky^2 = 0$, then $k =$

(a) 7 (b) 14 (c) -14 (d) -7

7. If one angular bisector of $a(x + 1)^2 + 2h(x + 1)(y - 3) + b(y - 3)^2 = 0$ is $2x - 3y + 11 = 0$, then equation of other angular bisector is

(a) $x + y = 2$ (b) $3x + 2y - 11 = 0$

(c) $3x + 2y = 0$ (d) $3x + 2y - 3 = 0$

8. The lines $2x + 3y = 6$, $2x + 3y = 8$ cut the x-axis at A and B. A line 'l' is drawn through the point (2, 2) meets the x-axis at C in such a way that the abscissa of A, B and C are in geometric progression. The equation of the line l is

(a) $3x - 5y = 16$ (b) $3x + 5y = 16$

(c) $5x + 3y = 16$ (d) $3x - 3y = 16$

9. Two sides of a rhombus $ABCD$ are parallel to the lines $y = x + 2$ and $y = 7x + 3$. If the diagonals intersect at (1, 2) and the vertex $A = (0, a)$ where $a \neq 0$, then 'a' is equal to

(a) 2 (b) $-1/2$ (c) 4 (d) 5/2

10. If one side of a rhombus has end point (4, 5) and (1, 1), then maximum area of the rhombus is

(a) 5 (b) 25 (c) 12.5 (d) 61/2

11. Let ABC be an equilateral triangle with $AB = 6$, and vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3), then $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2$ equals

(a) 192 (b) 243 (c) 486 (d) 972

12. The square $ABCD$ where $A(0, 0)$, $B(2, 0)$, $C(2, 2)$, $D(0, 2)$ undergoes the following transformations successively

(i) $f_1(x, y) = (y, x)$ (ii) $f_2(x, y) = (x + 3y, y)$

(iii) $f_3(x, y) = \left(\frac{x-y}{2}, \frac{x+y}{2}\right)$. Then the final figure is

(a) a square (b) a rectangle

(c) a parallelogram (d) a rhombus

13. The equation of the line belonging to the family of the lines given by $3x + 4y + 6 + \lambda(x + y + 2) = 0$ (where λ being a parameter) and situated at the greatest distance from the point (2, 3) is

(a) $4x - 3y + 8 = 0$ (b) $4x + 3y + 8 = 0$

(c) $4x + 3y - 8 = 0$ (d) None of these

14. If (0, 0), $(a, 2)$, $(2, b)$ form the vertices of an equilateral triangle, where a and b not lie between 0 and 2, then the value of $4(a + b) - ab$ equals

(a) 4 (b) -4 (c) 2 (d) 8

15. The number of integral values of m for which the abscissa of the point of intersection of the lines $5x + 3y = 87$ and $y = mx + 1$ is also an integer is

(a) 2 (b) 3

(c) 4 (d) None of these

16. Vertices of a triangle are (4, 3), $(5\sin\theta, 5\cos\theta)$, $(5\cos\theta, -5\sin\theta)$. Then the locus of the orthocentre is

(a) a straight line (b) a circle

(c) ellipse (d) hyperbola

17. If the area of triangle formed by the points $(2a, b)$, $(a + b, 2b + a)$ and $(2b, 2a)$ be λ, then the area of the triangle whose vertices are $(a + b, a - b)$, $(3b - a, b + 3a)$ and $(3a - b, 3b - a)$ will be

(a) $\frac{3}{2}\lambda$ (b) 3λ

(c) 4λ (d) None of these

18. The vertices of a triangle are $A(x_1, x_1\tan\alpha)$, $B(x_2, x_2\tan\beta)$ and $C(x_3, x_3\tan\gamma)$. If the circumcentre of ΔABC coincides with the origin and $H(a, b)$ be its orthocentre, then a/b is equal to

(a) $\dfrac{\cos\alpha+\cos\beta+\cos\gamma}{\cos\alpha\cos\beta\cos\gamma}$ (b) $\dfrac{\sin\alpha+\sin\beta+\sin\gamma}{\sin\alpha+\sin\beta\sin\gamma}$

(c) $\dfrac{\tan\alpha+\tan\beta+\tan\gamma}{\tan\alpha\ \tan\beta\ \tan\gamma}$ (d) $\dfrac{\cos\alpha+\cos\beta+\cos\gamma}{\sin\alpha+\sin\beta+\sin\gamma}$

19. The image of $P(a, b)$ on $y = -x$ is Q and the image of Q on the line $y = x$ is R. Then the mid point of PR is

(a) $(a + b, b + a)$ (b) $\left(\dfrac{a+b}{2}, \dfrac{b+a}{2}\right)$

(c) $(a - b, b - a)$ (d) $(0, 0)$

20. A straight oine L with negative slope passes through the point (8, 2) and cuts the positive coordinate axes at points P and Q. As L varies, the absolute minimum value of $OP + OQ$ is (O is origin)

(a) 10 (b) 18 (c) 16 (d) 12

21. The line $3x - 4y + 7 = 0$ is rotated thorugh an angle $\pi/4$ in the clockwise direction about the point $(-1, 1)$. The equation of the line in its new position is

(a) $7y + x - 6 = 0$ (b) $7y - x - 6 = 0$

(c) $7y + x + 6 = 0$ (d) $7y - x + 6 = 0$

SECTION-II

More than One Correct Answer Type

22. If the lines $x + y + 4 = 0$, $x - y + 6 = 0$ and $x + ky + k_1 = 0$ do not form a triangle, then the value of k is (if k_1 is variable)

(a) $k = 1$ (b) $k = -1$

(c) $k = 2$ (d) $k = 3$

23. Two lines $x - y - 6 = 0$ and $x + y + 4 = 0$ divide the xy-plane into four quadrants. If (0,0) lies the first quadrant, then

(a) $(1, 1) \in$ I (b) $(1, 1) \in$ II

(c) $(-3, -3) \in$ II (d) $(-3, -3) \in$ III

24. Two straight lines $u = 0$ and $v = 0$ pass through the origin and angle between them is $\tan^{-1}(7/9)$. If the ratio of the slope of $v = 0$ and $u = 0$ is 9/2, then their equations are

(a) $y + 3x = 0$ and $3y + 2x = 0$

(b) $2y + 3x = 0$ and $3y + x = 0$,

(c) $2y = 3x$ and $3y = x$

(d) $y = 3x$ and $3y = 2x$

25. The combined equation of three sides of a triangle is $(x^2 - y^2)(2x + 3y - 6) = 0$. If $(-2, a)$ is an interior points and $(b, 1)$ is an exterior point of the triangle, then

(a) $2 < a < 10/3$ (b) $-2 < a < 10/3$

(c) $-1 < b < 9/2$ (d) $-1 < b < 1$

26. Two sides of a rhombus $OABC$ (lying entriely in first quadrant or third quadrant) of area equal to 2 sq. units are $y = x/\sqrt{3}, y = \sqrt{3}\ x$. Then possible coordinates of B is/are (O being origin)

(a) $(1+\sqrt{3}, 1+\sqrt{3})$ (b) $(-1-\sqrt{3}, -1-\sqrt{3})$

(c) $(3+\sqrt{3}, 3+\sqrt{3})$ (d) $(\sqrt{3}-1, \sqrt{3}-1)$

27. The coordinates of three points O, A and B are (0, 0), (0, 4) and (6, 0), respectively. If a point P moves so that the area of ΔPOA is always twice the area of ΔPOB, then P lies on

(a) $x - 3y = 0$ (b) $x + 3y = 0$

(c) $3x + 4y = 0$ (d) $3x - 4y = 0$

28. The point of intersection of the lines $\dfrac{x}{a}+\dfrac{y}{b}=1$ and $\dfrac{x}{b}+\dfrac{y}{a}=1$ lies on

(a) $x - y = 0$ (b) $(x + y)(a + b) = 2ab$

(c) $(lx + my)(a + b) = (l + m)ab$

(d) $(lx - my)(a + b) = (l - m)ab$

29. The equations $(b - c)x + (c - a)y + a - b = 0$, $(b^3 - c^3)x + (c^3 - a^3)y + a^3 - b^3 = 0$ will represent the same line if

(a) $b = c$ (b) $c = a$

(c) $a = b$ (d) $a + b + c = 0$

30. The area of a triangle is 5 sq unit. Two of its vertices are (2, 1) and (3, –2). The third vertex lies on $y = x + 3$. The coordinates of the third vertex can be

(a) (–3/2, 3/2) (b) (3/4, –3/2)

(c) (7/2, 13/2) (d) (–1/4, 11/4)

31. If the point $P(x, y)$ be equidistant from the points $A(a + b, a - b)$ and $B(a - b, a + b)$, then

(a) $ax = by$ (b) $bx = ay$

(c) $x^2 - y^2 = 2(ax + by)$

(d) P is the mid point of AB

32. If the lines $x - 2y - 6 = 0$, $3x + y - 4 = 0$ and $\lambda x + 4y + \lambda^2$ are concurrent, then

(a) $\lambda = 2$ (b) $\lambda = -3$

(c) $\lambda = 4$ (d) $\lambda = -4$

33. Equation of a straight line passing through the point of intersection of $x - y + 1 = 0$ and $3x + y - 5 = 0$ are perpendicular to one of them is

(a) $x + y + 3 = 0$ (b) $x + y - 3 = 0$

(c) $x - 3y - 5 = 0$ (d) $x - 3y + 5 = 0$

34. If the lines $ax + by + c = 0$, $bx + cy + a = 0$ and $cx + ay + b = 0$ are concurrent $(a + b + c \neq 0)$, then

(a) $a^3 + b^3 + c^3 - 3abc = 0$

(b) $a = b$ (c) $a = b = c$

(d) $a^2 + b^2 + c^2 - bc - ca - ab = 0$

35. The slope of the straight line which is both tangent and normal to the curve $4x^3 = 27y^2$ is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{-1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) $-\sqrt{2}$

SECTION-III

Comprehension Type

Paragraph for Question No. 36 to 37

If $3x^2 + 8xy - 3y^2 = 0$ represents a pair of lines AB and BC where as the equation $3x^2 + 8xy - 3y^2 + 2x - 4y - 1 = 0$ represents the lines CD and DA, then

36. The possible equation of CD

(a) $x + y + 1 = 0$ (b) $x + 3y + 1 = 0$

(c) $2x - 4y - 1 = 0$ (d) $3x - y = 0$

37. Area of quadrilateral $ABCD$

(a) 1/10 (b) 1/5 (c) 1/20 (d) 1/15

Paragraph for Question No. 38 to 40

Let $A(0, \beta)$, $B(-2, 0)$ and $C(1, 1)$ be the vertices of a triangle, then

38. Angle A of the triangle ABC will be obtuse if β lies in

(a) $(-1, 2)$ (b) $\left(2, \frac{5}{2}\right)$

(c) $\left(-1, \frac{2}{3}\right) \cup \left(\frac{2}{3}, 2\right)$ (d) none of these

39. If l_1 is the interval of values of β for which A is obtuse and l_2 be the inteval of values of β for which A is largest angle of ΔABC, then

(a) $l_1 = l_2$ (b) l_1 is subset of l_2

(c) l_2 is a subset of l_1 (d) none of these

40. All the values of β for which angle A of the triangle ABC is largest lie in interval :

(a) $(-2, 1)$ (b) $\left(-2, \frac{2}{3}\right) \cup \left(\frac{2}{3}, 1\right)$

(c) $\left(-2, \frac{2}{3}\right) \cup \left(\frac{2}{3}, \sqrt{6}\right)$ (d) none of these

Paragraph for Question No. 41 to 43

For points $P \equiv (x_1, y_1)$ and $Q \equiv (x_2, y_2)$ of the coordinate plane, a new distance $d(P, Q)$ is defined by $d(P, Q) = |x_1 - x_2| + |y_1 - y_2|$

Let $O = (0, 0)$, $A \equiv (1, 2)$, $B \equiv (2, 3)$ and $C \equiv (4, 3)$ are four fixed points on xy-plane.

41. Let $R(x, y)$, such that R is equidistant from the points O and A with respect to new distance and if $0 \leq x < 1$, $0 \leq y < 2$, then R lie on a line segment whose equation is

(a) $x + y = 3$ (b) $x + 2y = 3$

(c) $2x + y = 3$ (d) $2x + 2y = 3$

42. Let $S(x, y)$, such that S is equidistant from points O and B with respect to new distance and if $x \geq 2$ and $0 \geq y < 3$, then locus of S is

(a) a line segment of finite length

(b) a line of infinite length

(c) a ray of finite length

(d) a ray of infinite length

43. Distance between the circumcentre and orthocentre of ΔABC is

(a) 5 (b) $\sqrt{29}$ (c) 7 (d) $\sqrt{37}$

Paragraph for Question No. 44 to 46

The locus of a moving point is the path traced out by that point under one or more given condition. Technically, a locus represents the 'set of all points' which lies on it.

A relation $f(x, y) = 0$ between x and y which is satisfied by each point on the locus and such that each point satisfying the equation is on the locus is called the equation of the locus.

44. Locus of centroid of the triangle whose vertices are $(a\cos t, a\sin t)$, $(b\sin t, -b\cos t)$ and $(1, 0)$, where t is a parameter, is

(a) $(3x - 1)^2 + (3y)^2 = a^2 - b^2$

(b) $(3x - 1)^2 + (3y)^2 = a^2 + b^2$

(c) $(3x + 1)^2 + (3y)^2 = a^2 + b^2$
(d) $(3x + 1)^2 + (3y)^2 = a^2 - b^2$

45. Let $A(2, -3)$ and $B(-2, 1)$ be vertices of a ΔABC. If the centroid of this triangle moves on the line $2x + 3y = 1$, then the locus of the vertex C is the line

(a) $2x + 3y = 9$ (b) $2x - 3y = 7$
(c) $3x + 2y = 5$ (d) $3x - 2y = 3$

46. If the equation of the locus of a point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$, then the value of '$c$' is

(a) $\frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$

(b) $a_1^2 - a_2^2 + b_1^2 - b_2^2$

(c) $\frac{1}{2}(a_1^2 + a_2^2 + b_1^2 + b_2^2)$

(d) $\sqrt{(a_1^2 + b_1^2 - a_2^2 - b_2^2)}$

SECTION-IV

Matrix-Match Type

47. The family of straight lines $ax + by + c = 0$ passing through a fixed point (s) . Given that relation between a, b and c :

Column I		Column II	
(A)	a, b, c are in A.P.	(p)	$(-2, -3)$
(B)	$4a^2 + 9b^2 + 12ab = c^2$	(q)	$(2, 3)$
(C)	$a^2 + 4b^2 - 4ab = c^2$	(r)	$(-1, 2)$
(D)	$2a + 3b - c = 0$	(s)	$(1, -2)$

48. Match the following:

Column I		Column II	
(A)	The distance between the lines $(x+7y)^2 + 4\sqrt{2}(x+7y) - 42 = 0$ is	(p)	2
(B)	If the sum of the distances of a point from two perpendicular lines in a plane is 1, then its locus is $\|x\| + \|y\| = k$, where k is equal to	(q)	7
(C)	If $6x + 6y + m = 0$ is acute angle bisector of line $x + 2y + 4 = 0$ and $4x + 2y - 1 = 0$, then m is equal to	(r)	3
(D)	Area of the triangle formed by the lines $y^2 - 9xy + 18x^2 = 0$ and $y = 6$ is	(s)	1

49. Match the following:

Column I		Column II	
(A)	The family of lines $x(2a + 3b) + y(3a + 2b) = 2(b - a)$ passes through the point for all values of a and b, then the point is	(p)	$(3, -2)$
(B)	If $9a^2 + 4b^2 - c^2 + 12ab = 0$, then the family of st. lines $ax + by + c = 0$ is concurrent at	(q)	$(3, 2)$
(C)	If $12a^2 - 6b^2 - c^2 + ab + 5bc + ac = 0$, then the family of lines $ax + by + c = 0$ is concurrent at	(r)	$(-4, -3)$
		(t)	$(-3, -2)$

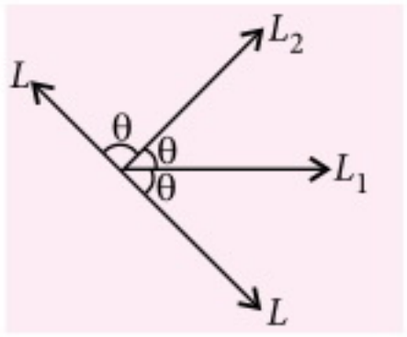
50. Match the following:

Column I		Column II	
(A)	For the lines $4x + 3y - 6 = 0$ and $5x + 12y + 9 = 0$, acute angle bisector and obtuse angle bisectors represented by A and O respectively, then	(p)	$A : 7x - 9y - 3 = 0$
(B)	For the lines $4x - 3y - 6 = 0$ and $5x - 12y + 9 = 0$, acute angle bisector and obtuse angle bisectors represented by A and O respectively, then	(q)	$A : 7x - 9y + 3 = 0$
(C)	For the straight lines $4x - 3y + 6 = 0$ and $5x - 12y - 9 = 0$, acute angle bisector and obtuse angle bisectors represented by A and O respectively, then	(r)	$A : 7x + 9y - 3 = 0$
		(s)	$O : 9x + 7y - 41 = 0$
		(t)	$O : 9x - 7y - 41 = 0$

SOLUTION

1. (b) : $m = \tan(45° + \theta) = 3$ where $\tan\theta = 1/2$

2. (c) : $3\theta = \pi$



3. (b) : Let $OP = a'$, $OQ = b'$

$\Rightarrow\ a - a' = b - b'$ (i)

Equation of $AQ : \frac{x}{a} + \frac{y}{b'} = 1$;

Equation of $PB : \frac{x}{a'} + \frac{y}{b} = 1 \Rightarrow \frac{x}{aa'} + \frac{y}{bb'} = 0$

(using (i))

Also $\frac{x}{ab} + \frac{y}{bb'} = \frac{1}{b}, \frac{x}{aa'} + \frac{y}{ab} = \frac{1}{a} \Rightarrow \frac{x+y}{ab} = \frac{a+b}{ab}$

4. (b)

5. (c) : $ax^2 + 2hxy + by^2 = 0$ represent two sides of triangle and (c, d) is orthocentre, then equation of third side is $(a + b)(cx + dy) = ad^2 - 2hcd + bc^2$

$3(2x + y) = 1 - (-3)2 + 2 \cdot 4 \Rightarrow 2x + y = 5$

6. (b) : Equation of angle bisector of pair of lines $9x^2 + 12xy + ky^2 = 0$ is $6(x^2 - y^2) - xy(9 - k) = 0$

Bisector is parallel to $2x + 3y = 7$ and passing through origin

$\Rightarrow\ y = -\frac{2x}{3}$

7. (d) : Angle bisectors are perpendicular.

So, equation of other bisector $3x + 2y + k = 0$

Shifting origin to $(-1, 3)$,we get other line equation $3x + 2y = 3$

8. (b) : L_1: $2x + 3y - 6 = 0 \Rightarrow A(3, 0)$

$L_2 : 2x + 3y - 8 = 0 \Rightarrow B(4, 0), r = \frac{4}{3}, a = 3$

$ar^2 = C\left(\frac{16}{3}, 0\right)$

Equation of line through $(2, 2)$ and $\left(\frac{16}{3}, 0\right)$ is

$3x + 5y - 16 = 0.$

9. (d) : Daigonals are parallel to the angle bisectors of given lines

$$\frac{x - y + 2}{\sqrt{2}} = \pm\frac{7x - y + 3}{5\sqrt{2}}$$

$5x - 5y + 10 = 7x - y + 3$ or $-7x + y - 3$

$-2x - 4y + 7 = 0$ or $12x - 6y + 13 = 0$

Equation of one diagonal $2x + 4y + k = 0$ passing through $(1, 2) \Rightarrow k = -10$ intersect y-axis at $(0, 5/2)$

10. (b)

11. (d) : $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = 4\ (\text{area of } \Delta ABC)^2$

$= 4\left(\frac{\sqrt{3}}{4}(36)\right)^2 = 972$ sq. units

12. (c) : $f_3 \cdot f_2 \cdot f_1(x, y) = f_3 \cdot f_2(y, x) = f_3(y + 3x, x)$

$= \left(\frac{y + 3x - x}{2}, \frac{y + 3x + x}{2}\right) = \left(\frac{2x + y}{2}, \frac{4x + y}{2}\right)$

$\therefore\ A\ (0, 0) \to A'|(0,0),\ B\ (2, 0) \to B'|\ (2, 4),$
$C\ (2, 2) \to C'(3, 5), D(0, 2) \to D'(1, 1)$

$\therefore$ It is easily seen $A'B' = D'\ C', A'D' = B'C',$
$A'B' \neq B'C'.$

Hence, it is neither a square nor a rhombus.

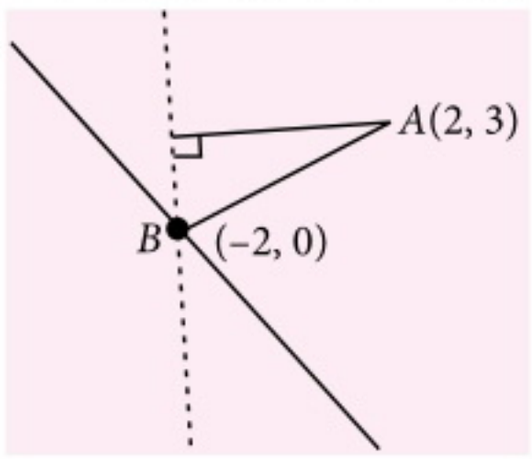
Further slope of $A'B' \times A'D' = \frac{4}{2} \cdot \frac{1}{-1} \neq -1$

Hence AB is not perpendicular to AD.

$\Rightarrow$ It is not a rectangle

$\therefore$ $ABCD$ is a parallelogram.

13. (b) : The family of lines $3x + 4y + 6 + \lambda(x + y + 2)$ are concurrent at the intersection of $3x + 4y + 6 = 0$ and $x + y + 2 = 0$ *i.e.*, at $(-2, 0)$



$\therefore$ the line through $(-2, 0)$ lying at the greatest distance from $A(2,3)$ is the line through $B(-2, 0)$ perpendicular to AB.

Hence its equation is

$$y - 0 = -\frac{4}{3}(x + 2)\ \ i.e., 4x + 3y + 8 = 0$$

14. (a) : ΔABC, where $A(0, 0)$, $B(a, 2)$, $C(2, b)$ is a equilateral.

$\Rightarrow \quad AB = AC = BC$

$\Rightarrow \quad a^2 + 4 = b^2 + 4 = (a-2)^2 + (b-2)^2$

$\Rightarrow \quad a^2 = b^2$ and $b^2 - 4a - 4b + 4 = 0$

If $a = -b$, $b^2 - 4a - 4b + 4 \Rightarrow b^2 + 4 = 0$

If $a = b$, then $b^2 - 8b + 4 = 0 \quad \Rightarrow \quad b = 4 \pm 2\sqrt{3} = a$

Let $a = b = 4 + 2\sqrt{3}$

$\therefore \quad 4(a+b) - ab = 4(8 + 4\sqrt{3}) - (4 + 2\sqrt{3})^2 = 4$

Similarly,

$a = b = 4 - 2\sqrt{3} \quad \Rightarrow \quad 4(a + b) - ab = 4$

15. (d) : Solving the equation $5x + 3y = 87$, $y = mx + 1$ for x we get $x = \dfrac{84}{5+3m}$.

The factor of 84 are $\pm 1, +2, \pm 3, \pm 4, \pm 6, \pm 7 \pm 12, \pm 14, \pm 21, \pm 28, \pm 48, \pm 84$.

16. (B) : Let $A\ (4, 3)$, $B(5\sin\theta, 5\cos\theta)$, $C(5\cos\theta, -5\sin\theta)$ be the vertices and G, H be the centroid and orthocentre of ΔABC. If O is the origin $OA^2 = 25 = OB^2 = OC^2$.

$\therefore \quad O$ is the circumcentre of ΔABC. G divides OH in the ratio 1 : 2.

Hence $\dfrac{4 + 5\sin\theta + 5\cos\theta}{3} = \dfrac{x_1}{3}$

$\dfrac{3 + 5\cos\theta - 5\sin\theta}{3} = \dfrac{y_1}{3}$

H is $(4 + 5\sin\theta + 5\cos\theta, 3 + 5\cos\theta - 5\sin\theta) = (x_1, y_1)$

H is $(x_1, y_1) \quad \Rightarrow \quad (x_1 - 4)^2 + (y_1 - 3)^2 = 50$

$\Rightarrow$ locus of H is $(x-4)^2 + (y-3)^2 = 50$ which is a circle.

17. (c) : We know that the area of the triangle formed by joining the mid points of any triangle is one fourth of that triangle

18. (d) : Let R be the radius of the circumcircle and O be the origin, then $AO = \sqrt{(x_1^2 + x_1^2 \tan^2\alpha)}$

$\Rightarrow \quad R = x_1 \sec\alpha \quad \Rightarrow \quad x_1 = R\cos\alpha$

Similarly, $x_2 = R\cos\beta$ and $x_3 = R\cos\gamma$

So, the coordinates of vertices are $A(R\cos\alpha, R\sin\alpha)$, $B(R\cos\beta, R\sin\beta)$, $C(R\cos\gamma, R\sin\gamma)$.

Hence, the coordinates of centroid G are

$$\left(\frac{\Sigma R\cos\alpha}{3}, \frac{\Sigma R\sin\alpha}{3}\right)$$

Since, the orthocentre $H(a, b)$, circumcentre $O(0, 0)$ and the centroid G are collinear

$\therefore \quad$ slope of OH = slope of OG

$$\Rightarrow \quad \frac{b}{a} = \frac{R(\sin\alpha + \sin\beta + \sin\gamma)}{R(\cos\alpha + \cos\beta + \cos\gamma)}$$

$$\therefore \quad \frac{a}{b} = \frac{\cos\alpha + \cos\beta + \cos\gamma}{\sin\alpha + \sin\beta + \sin\gamma}$$

19. (d) : The image of $P(a, b)$ on $y = -x$ is $Q(-b, -a)$ (interchange and change signs) and the image of $Q(-b, -a)$ on $y = x$ is $R(-a, -b)$ (merely interchange)

$\therefore \quad$ The mid point of PR is $(0, 0)$

20. (b) : The equation of the line L be

$y - 2 = m(x - 8)$, $m < 0$

Coordinates of P and Q are

$P\left(8 - \dfrac{2}{m}, 0\right)$ and $Q(0, 2 - 8m)$.

So, $OP + OQ = 8 - \dfrac{2}{m} + 2 - 8m$

$$= 10 + \frac{2}{(-m)} + 8(-m) \geq 10 + 2\sqrt{\frac{2}{(-m)} \times 8(-m)} \geq 18$$

So, absolute minimum value of $OP + OQ = 18$

21. (a) : As $(-1, 1)$ is a point on $3x - 4y + 7 = 0$, the rotation is possible

Slope of the given line = 3/4.

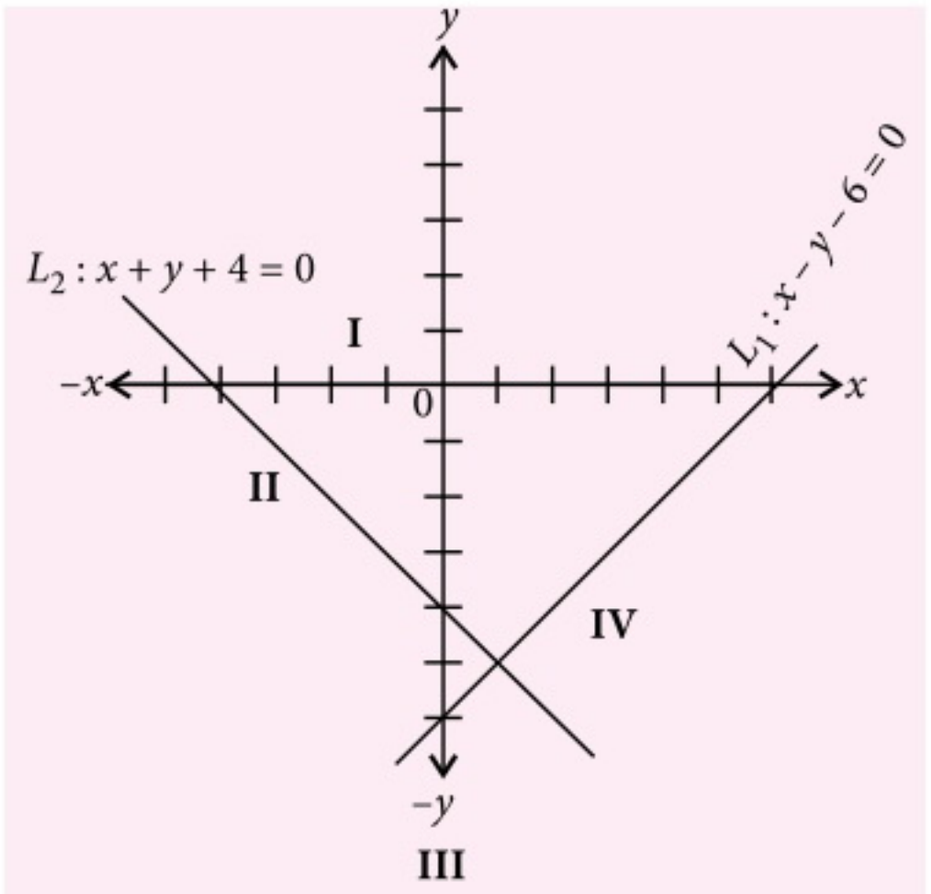
Slope of the line in its new position $= \dfrac{\frac{3}{4} - 1}{1 + \frac{3}{4}} = -\dfrac{1}{7}$

The required equation is $y - 1 = -\dfrac{1}{7}(x+1)$

or $\quad 7y + x - 6 = 0$

22. (a, b) : $x + ky + k_1 = 0$, will be parallel to either $x + y + 4 = 0$ or $x - y + 6 = 0$; hence $k = 1$, $k = -1$.

23. (a, c) :



24. (a, b, c, d) : Let the slope of $u = 0$ be m.

Then slope of $v = 0$ is $\frac{9m}{2}$.

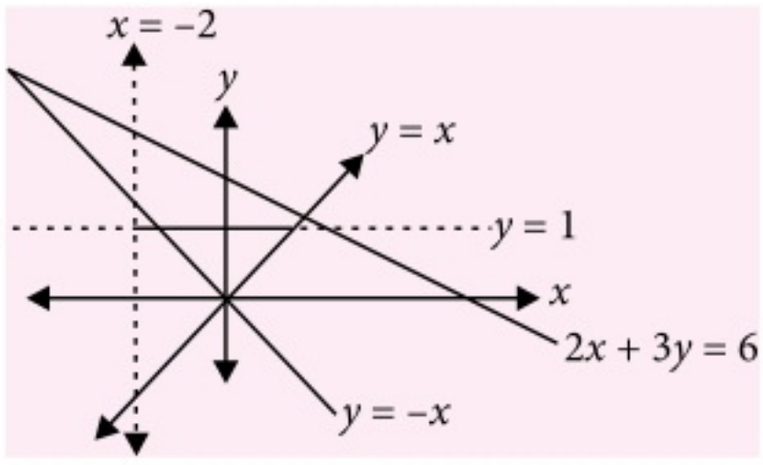
$$\text{Now, } \frac{7}{9} = \left|\frac{m - \frac{9m}{2}}{1 + m \times \frac{9m}{2}}\right| = \left|\frac{-7m}{2 + 9m^2}\right|$$

$$\Rightarrow 9m^2 - 9m + 2 = 0 \text{ or } 9m^2 + 9m + 2 = 0$$

$$m = \frac{9 \pm \sqrt{81 - 72}}{18} = \frac{9 \pm 3}{18} = \frac{2}{3}, \frac{1}{3}$$

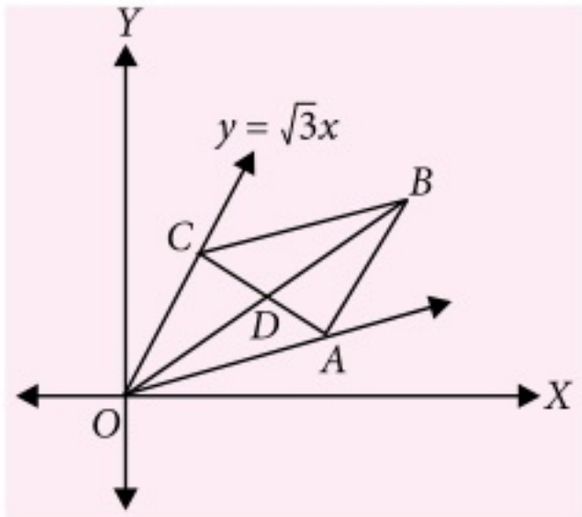
$$\text{or, } m = \frac{-9 \pm 3}{18} = -\frac{2}{3}, -\frac{1}{3}$$

25. (a, d) :



The separate equations of the sides are $x - y = 0$, $x + y = 0$ and $2x + 3y - 6 = 0$. The point $(-2, a)$ moves on the line $x = -2$ and $(b, 1)$ moves on the line $y = 1$. From the figure, the Y-coordinates of points of intersection of $x = -2$ with $y = -x$ and $2x + 3y = 6$ give the range of values of a. The X-coordinates of the points of intersection of $y = 1$ with $y = -x$ and $y = x$ give the range of values of b.

26. (a, b) : Here $\angle COA = 90°$



$$\text{Area of rhombus} = 2 \times \frac{1}{2} \times OA \times OC \sin 30°$$

$$\Rightarrow 2 = \frac{1}{2}x^2 \Rightarrow OA = OC = 2$$

Also, $\angle OAB = 150°$

$$\Rightarrow \cos 150° = \frac{OA^2 + AB^2 - OB^2}{2\lambda OA \times AB}$$

$$OB^2 = 8 + 4\sqrt{3} \Rightarrow OB = 2(4 + 2\sqrt{3})$$

$$= (\sqrt{2}(\sqrt{3} + 1))^2 = \sqrt{2}(\sqrt{3} + 1)$$

Hence, the coordinates of B are

$$(\pm\sqrt{2}(\sqrt{3} + 1)\cos 45°, \ \pm\sqrt{2}(\sqrt{3} + 1)\sin 45°)$$

27. (a, b) : Let $P(x, y)$ be the moving point. Then the area of ΔPOA is $\frac{1}{2}|x \cdot 4 - y \cdot 0| = |2x|$, and that of ΔPOB is $\frac{1}{2}|x \cdot 0 - y \cdot 6| = |-3y|$, because one vertex of the triangle is at the origin. Therefore, from the given condition, we have

$$|2x| = 2|-3y| \text{ or } x = \pm(3y)$$

$$\Rightarrow x - 3y = 0 \text{ or } x + 3y = 0$$

28. (a, b, c, d) : Point of intersection of $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{b} + \frac{y}{a} = 1$ is $P\left(\frac{ab}{a+b}, \frac{ab}{a+b}\right)$, this point P satisfies alternatives (a), (b), (c) and (d)

29. (a, b, c, d) : Two lines will be identical if their exists some real number k such that

$$b^3 - c^3 = k(b - c), \ c^3 - a^3 = k(c - a)$$

and $a^3 - b^3 = k(a - b)$

$$\Rightarrow b - c = 0 \text{ or } b^2 + c^2 + bc = k$$

$$c - a = 0 \text{ or } c^2 + a^2 + ca = k$$

and $a - b = 0$ or $a^2 + b^2 + ab = k$

$$\Rightarrow a = b \text{ or } b = c \text{ or } c = a$$

$$\text{or, } a = b \text{ or } a + b + c = 0$$

30. (a, b) : As the third vertex lies on the line $y = x + 3$, its coordinates are of the form $(x, x + 3)$. The area of the triangle with vertices $(2, 1)$, $(3, -2)$ and $(x, x + 3)$ is given by

$$\frac{1}{2}\begin{vmatrix} x & x+3 & 1 \\ 2 & 1 & 1 \\ 3 & -2 & 1 \end{vmatrix} = |2x-2| = 5 \qquad \text{(given)}$$

$\Rightarrow \quad x = -3/2, 7/2$

Thus, the coordinates of the third vertex are $(7/2, 13/2)$ or $(-3/2, 3/2)$

31. (b, d) : It is clear $A(a + b, a - b)$ is the image of $B(a - b, a + b)$ w.r.t $y = x$

$\therefore \quad P$ is the mid point of AB *i.e.*, (a, a)

If P be (a, b), then $bx = ay$ ($\because \ a = b$)

32. (a, d) : $\begin{vmatrix} 1 & -2 & -6 \\ 3 & 1 & -4 \\ \lambda & 4 & \lambda^2 \end{vmatrix} = 0$

$\Rightarrow \quad \lambda^2 + 2\lambda - 8 = 0$

$\therefore \quad (\lambda + 4)(\lambda - 2) = 0 \quad \Rightarrow \lambda = -4, 2$

33. (a, d) : Equation of any line through the point of intersection of the given lines is $(3x + y - 5) + \lambda(x - y + 1) = 0$.

Since this line is perpendicular to one of the given lines $\left(\frac{3+\lambda}{\lambda-1} \times 1\right) = -1$ or $\frac{3+\lambda}{\lambda-1} \times (-3) = -1$

$\Rightarrow \quad \lambda = -1$ or -5,

Therefore the required straight line is $x + y - 3 = 0$ or $x - 3y + 5 = 0$

34. (a, c, d) : Since $ax + by + c = 0$, $bx + cy + a = 0$ and $cx + ay + b = 0$ are concurrent

$$\therefore \quad \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$$

$\Rightarrow \quad 3abc - a^3 - b^3 - c^3 = 0$

$\Rightarrow \quad -(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

as $a + b + c \neq 0$

$\therefore \quad a^2 + b^2 + c^2 - ab - bc - ca = 0$

$$\frac{1}{2}\{(a-b)^2 + (b-c)^2 + (c-a)^2\} = 0$$

As a, b, c are real numbers

$\therefore \quad b - c = 0, c - a = 0, a - b = 0$

$\Rightarrow \quad a = b = c$

35. (c, d): Let $\frac{x^3}{27} = \frac{y^2}{4} = t^6$

$\Rightarrow \quad x = 3t^2, \ y = 2t^3 \quad \Rightarrow \quad \frac{dy}{dx} = t$

Tangent at 't' is $tx - y = t^3$ ... I

Normal at 't_1' is $t_1 y + x = 2t_1^4 + 3t_1^2$ ... II

I, II are identical, $\frac{t}{1} = \frac{-1}{t_1} = \frac{t^3}{2t_1^4 + 3t_1^2}$

$\Rightarrow \quad -t^3 = 2t_1^3 + 3t_1, \ t_1 = \frac{-1}{t}$, eliminating t_1 we get

$t^6 = 2 + 3t^2 \quad \Rightarrow \ t^2 = 2 \ \Rightarrow \ t = \pm\sqrt{2}$

36. (b) : $3x^2 + 8xy - 3y^2 = 0$

$(x + 3y)(3x - y) = 0 \quad \Rightarrow \ CD \parallel x + 3y$

$\Rightarrow$ Equation of CD is $x + 3y + 1 = 0$

37. (a) : Area of $ABCD = \left|\frac{-1}{\sqrt{6^2 + 8^2}}\right| = \frac{1}{10}$

38. (c) : Angle A will be obtuse if $BC^2 > AB^2 + AC^2$ and A, B, C are non collinear

$\Rightarrow \quad 10 > \beta^2 + 4 + 1 + (\beta - 1)^2$

$\Rightarrow \quad (\beta - 2)(\beta + 1) < 0 \quad \Rightarrow \quad \beta \in (-1, 2)$

But A, B, C are collinear for $\beta = 2/3$

$\Rightarrow$ Interval for β is $\left(-1, \frac{2}{3}\right) \cup \left(\frac{2}{3}, 2\right)$

39. (b) : Whenever A is obtuse it is the largest angle also. But there may be more values for which A is largest but is not obtuse.

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$\Rightarrow$ Choice (b) is correct.

40. (c) : Angle A will be largest if $a > b$, $a > c$, A, B, C are not collinear

$\Rightarrow a^2 > b^2, a^2 > c^2, \beta \neq \frac{2}{3}$

$\Rightarrow 10 > 1 - (\beta - 1)^2,\ 10 > 4 + \beta^2$

$\Rightarrow \beta^2 - 2\beta - 8 < 0,\ \beta^2 < 6$

$\Rightarrow -2 < \beta < 4, -\sqrt{6} < \beta < \sqrt{6}$

On taking intersection and rejecting $\beta = 2/3$, we get

$$\beta \in \left(-2, \frac{2}{3}\right) \cup \left(\frac{2}{3}, \sqrt{6}\right) \Rightarrow \text{(c) is correct.}$$

41. (d) : $\because$ $OR = AR$

$\Rightarrow |x - 0| + |y - 0| = |x - 1| + |y - 2|$

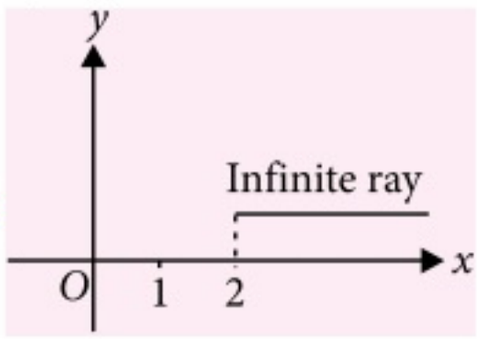
$\Rightarrow |x| + |y| = |x - 1| + |y - 2|$

$\because$ $0 \le x < 1$ and $0 \le y < 2$

$\therefore$ $x + y = -(x - 1) - (y - 2)$

$\Rightarrow 2x + 2y = 3$

42. (d) :



$\because$ $OS = BS$

$\Rightarrow |x - 0| + |y - 0| = |x - 2| + |y - 3|$

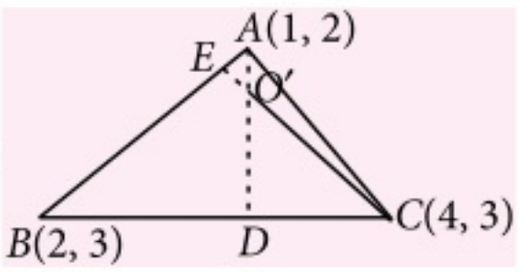
$\Rightarrow |x| + |y| = |x - 2| + |y - 3|$

$\because$ $x \ge 2$ and $0 \ge y < 3$

$\therefore$ $x + y = x - 2 + 3 - y \Rightarrow 2y = 1$

$\therefore$ $y = \frac{1}{2}$

43. (c) :



$\because$ $A \equiv (1, 2), B \equiv (2, 3)$ and $C \equiv (4, 3)$

$\therefore$ Centroid $G \equiv \left(\frac{7}{3}, \frac{8}{3}\right)$

Let $O'(\alpha, \beta)$ be the orthocentre

$\therefore$ Slope of $O'A \times$ Slope of $BC = -1$

$\Rightarrow \frac{\beta - 2}{\alpha - 1} \times \frac{0}{2} = -1$ or $\alpha - 1 = 0$

$\therefore$ $\alpha = 1$

Also, slope of $O'C \times$ Slope of $AB = -1$

$\Rightarrow \frac{\beta - 3}{\alpha - 4} \times \frac{3 - 2}{2 - 1} = -1$

$\Rightarrow \beta - 3 = -(\alpha - 4) = -(1 - 4) = 3$

$\therefore$ $\beta = 6$

$\therefore$ Orthocentre $O' \equiv (1, 6)$

$\because$ Centroid G divides orthocentre O' and circumcentre C' in the ratio 2 : 1 internally.

If $C' \equiv (x_1, y_1)$

$$O'(1, 6) \equiv G\left(\frac{7}{3}, \frac{8}{3}\right) \equiv C'(x_1, y_1)$$

$\therefore$ $\frac{7}{3} = \frac{2 \cdot x_1 + 1 \cdot 1}{2 + 1}$

$\Rightarrow x_1 = 3$ and $\frac{8}{3} = \frac{2 \cdot y_1 + 1 \cdot 6}{2 + 1}$

$\Rightarrow y_1 = 1.$ $\therefore$ $C' \equiv (3, 1)$

Then, $O'C' = |1 - 3| + |6 - 1| = 2 + 5 = 7$

44. (b) : Coordinate of a triangle are $(a\cos t, a\sin t)$, $(b\sin t, -b\cos t)$ and $(1, 0)$

$\therefore$ Centroid : $x = \frac{a\cos t + b\sin t + 1}{3}$

$\Rightarrow 3x - 1 = a\cos t + b\sin t$ ... (i)

and $y = \frac{a\sin t - b\cos t + 0}{3}$

$\Rightarrow 3y = a\sin t - b\cos t$ ... (ii)

Squaring and adding Eqs. (i) and (ii), we get

$(3x - 1)^2 + (3y)^2 = (a\cos t + b\sin t)^2 + (a\sin t - b\cos t)^2$

or, $(3x - 1)^2 + (3y)^2 = a^2 + b^2$

45. (a) : Let (x, y) be coordinate of vertex C and (x_1, y_1) be coordinates of centroid of the triangle

$\therefore$ $x_1 = \frac{x + 2 - 2}{3}$ and $y_1 = \frac{y - 3 + 1}{3}$

$\Rightarrow x_1 = \frac{x}{3}$ and $y_1 = \frac{y - 2}{3}$

$\because$ $2x_1 + 3y_1 = 1 \Rightarrow 2\left(\frac{x}{3}\right) + 3\left(\frac{y - 2}{3}\right) = 1$

$\Rightarrow 2x + 3y = 9$, which is the required locus of the vertex C.

46. (a) : Mid point of (a_1, b_1) and (a_2, b_2) is $\left(\frac{a_1 + a_2}{2}, \frac{b_1 + b_2}{2}\right)$

which lies on $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$

$$\Rightarrow\ (a_1-a_2)\frac{(a_1+a_2)}{2}+(b_1-b_2)\frac{(b_1-b_2)}{2}+c=0$$

$$\therefore\ c=\frac{1}{2}(a_2^2-a_1^2+b_2^2-b_1^2)\ =\frac{1}{2}(a_2^2+b_2^2-a_1^2-b_1^2)$$

47. A - s ; B - (p, q) ; C - (r, s) ; D - q

(A) $2b=a+c\ \Rightarrow\ a-2b+c=0\ \Rightarrow (1,-2)$

(B) $4a^2+9b^2+12ab=c^2\ \Rightarrow\ c=\pm(2a+3b)$

$\Rightarrow\ a(x\pm2)+b(y\pm3)=0\ \Rightarrow$ (2, 3) and (–2, –3)

(C) $a^2+4b^2-4ab=(a-2b)^2=c^2\ \Rightarrow\ c=\pm(a-2b)$

$\Rightarrow\ a(x\pm1)+b(y\pm2)=0\ \Rightarrow$ (–1, 2) and (1, –2)

(D) $2a+3b-c=0\ \Rightarrow\ c=2a+3b\ \Rightarrow$ (2, 3)

48. A - p ; B - s ; C - q ; D - r

(A) $(x+7y)^2+4\sqrt{2}(x+7y)-42=0$

$\Rightarrow\ (x+7y)^2+7\sqrt{2}(x+7y)-3\sqrt{2}(x+7y)-42=0$

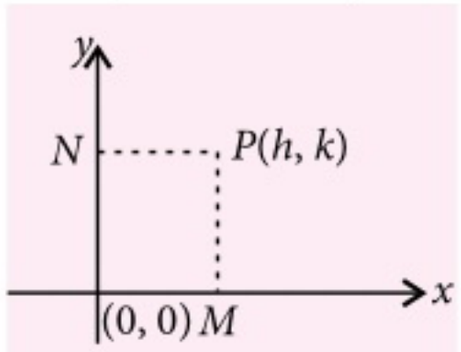
$\Rightarrow\ (x+7y)[x+7y+7\sqrt{2}]-3\sqrt{2}(x-7y+7\sqrt{2})=0$

$\Rightarrow\ (x+7y+7\sqrt{2})(x+7y-3\sqrt{2})=0$

$\Rightarrow\ x+7y+7\sqrt{2}=0$ and $x+7y-3\sqrt{2}=0$

$$\Rightarrow\ d=\left|\frac{7\sqrt{2}+3\sqrt{2}}{\sqrt{1+49}}\right|=\frac{10\sqrt{2}}{\sqrt{50}}=2$$

(B)



Let two perpendicular lines are coordinate axes.

Then, $PM+PN=1\ \Rightarrow\ h+k=1$

Hence, the locus is $x+y=1$.

But if the point lies in other quadrants also, then $|x|+|y|=1$.

Hence, value of k is 1.

(C) Angle bisectors between the lines $x+2y+4=0$ and $4x+2y-1=0$ is

$$\frac{x+2y+4}{\sqrt{1+4}}=\pm\frac{(-4x-2y+1)}{\sqrt{16+4}}$$

$$\Rightarrow\ x+2y+4=\pm\frac{(-4x-2y+1)}{2}$$

$$\Rightarrow\ 2(x+2y+4)=\pm(-4x-2y+1)$$

Since $AA'+BB'<0$, so +ve sign gives acute angle bisector. Hence,

$$2x+4y+8=-4x-2y+1$$

$\Rightarrow\ 6x+6y+7=0\ \Rightarrow\ m=7$

(D) We have, $y^2-9xy+18x^2=0$

or $y^2-6xy-3xy+18x^2=0$

$\Rightarrow\ y(y-6x)-3x(y-6x)=0$

$\Rightarrow\ (y-3x)=0$ and $(y-6x)=0$

The third line is $y=6$. Therefore, area of the triangle formed by these lines,

$$=\frac{1}{2}\begin{vmatrix}0&0&1\\1&6&1\\2&6&1\end{vmatrix}=\frac{1}{2}|6-12|=3\text{ units}^2$$

49. A - s ; B - (q, t) ; C - (p, r)

(A) $\because\ x(2a+3b)+y(3a+2b)=2(b-a)$

$\Rightarrow\ a(2x+3y+2)+b(3x+2y-2)=0$

Which is equation of a line passing through the point of intersection of the lines $2x+3y+2=0$ and $3x+2y-2=0$.

The point of intersection of these lines is (2, –2)

(B) Given $9a^2+4b^2-c^2+12ab=0$

or, $(3a+2b)^2-c^2=0$

or, $c=\pm(3a+2b)\ \Rightarrow\ 3a+2b+c=0$

or, $3a+2b-c=0\ \Rightarrow\ 3a+2b+c=0$

or, $-3a-2b+c=0$

and family of straight lines is $ax+by+c=0$

$\Rightarrow$ (3, 2) or (–3, –2) lies on line $ax+by+c=0$

(C) $12a^2+(b+c)a-(6b^2+c^2-5bc)=0$

$$\therefore\ a=\frac{-(b+c)\pm\sqrt{(b+c)^2+48(6b^2+c^2-5bc)}}{24}$$

$$=\frac{-(b+c)\pm(17b-7c)}{24}$$

$$=\frac{16b-8c}{24}\text{ or }\frac{-18b+6c}{24}\Rightarrow\frac{2b-c}{3}\text{ or, }\frac{-3b+c}{4}$$

or, $3a-2b+c=0$ or, $4a+3b-c=0$

$\Rightarrow$ (3, –2) or (–4, –3) lies on line $ax+by+c=0$

50. A - (r, t) ; B - (p, s) ; C - q

(A) $\because\ 4x+3y-6=0\ \Rightarrow\ -4x-3y+6=0$

and $5x+12y+9=0$

$\because\ (-4)(5)+(-3)(12)=-56<0$

$\therefore$ Bisectors are

$$\left(\frac{-4x-3y+6}{5}\right)=\pm\left(\frac{5x+12y+9}{13}\right)$$

$\Rightarrow\ (-52x-39y+78)=\pm(25x+60y+45)$

O : $(-52x - 39y + 78) = -(25x + 60y + 45)$

or $27x - 21y - 123 = 0$ or $9x - 7y - 41 = 0$

$\therefore$ $O : 9x - 7y - 41 = 0$

and A : $(-52x - 39y + 78) = +(25x + 60y + 45)$

or $77x + 99y - 33 = 0$ or $7x + 9y - 3 = 0$

$\therefore$ $A : 7x + 9y - 3 = 0$

(B) $\because$ $4x - 3y - 6 = 0 \implies -4x + 3y + 6 = 0$

and $5x - 12y + 9 = 0$

$\because$ $(-4)(5) + 3(-12) = -56 < 0$

$\therefore$ Bisectors are

$$\left(\frac{-4x+3y+6}{5}\right) = \pm\left(\frac{5x-12y+9}{13}\right)$$

$\implies$ $(-52x + 39y + 78) = \pm(25x - 60y + 45)$

O : $(-52x + 39y + 78) = -(25x - 60y + 45)$

or $27x + 21y - 123 = 0$ or $9x + 7y - 41 = 0$

$\therefore$ $O : 9x + 7y - 41 = 0$

and A : $(-52x + 39y + 78) = (25x - 60y + 45)$

or $77x - 99y - 33 = 0$ or $7x - 9y - 3 = 0$

$\therefore$ $A : 7x - 9y - 3 = 0$

(C) $\because$ $4x - 3y + 6 = 0$ and $5x - 12y - 9 = 0$

or $-5x + 12y + 9 = 0$

$\because$ $(4)(-5) + (-3)(12) = -56 < 0$

$\therefore$ Bisectors are

$$\left(\frac{4x-3y+6}{5}\right) = \pm\left(\frac{-5x+12y+9}{13}\right)$$

$\implies$ $(52x - 39y + 78) = \pm(-25x + 60y + 45)$

O : $(52x - 39y + 78) = -(-25x + 60y + 45)$

or $9x + 7y + 41 = 0$

$\therefore$ $O : 9x + 7y + 41 = 0$

and A : $(52x - 39y + 78) = (-25x + 60y + 45)$

or $77x - 99y + 33 = 0$ or $7x - 9y + 3 = 0$

$\therefore$ $A : 7x - 9y + 3 = 0$

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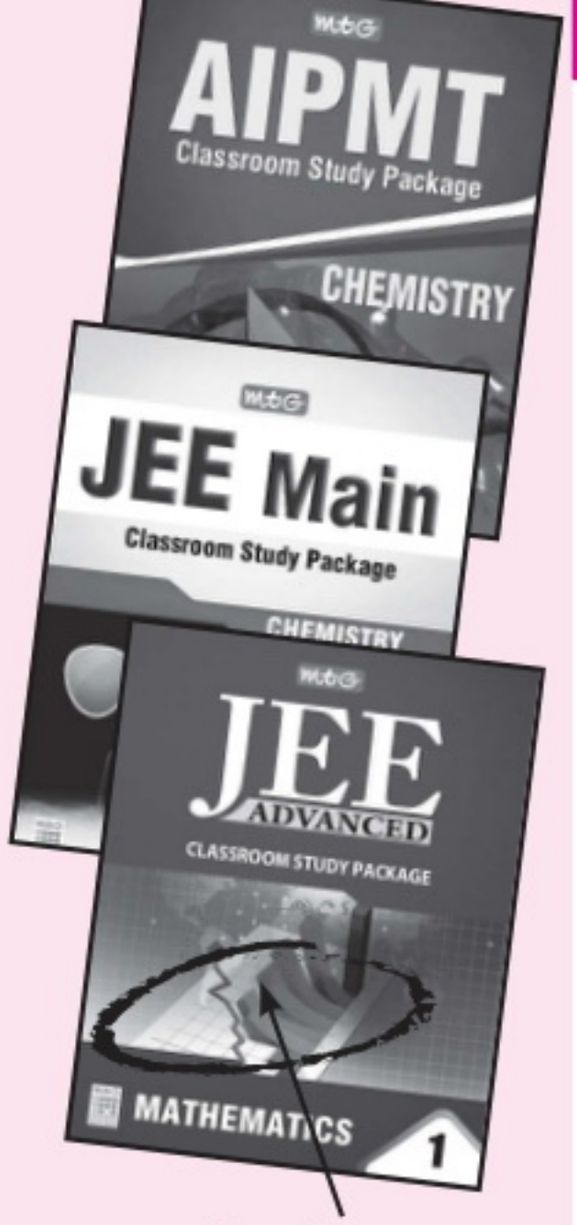

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concept BOOSTERS

Class XII

Integrals and Application of Integrals

* ALOK KUMAR, B.Tech, IIT Kanpur

This column is aimed at Class XII students so that they can prepare for competitive exams such as JEE Main/Advanced, etc and be also in command of what is being covered in their school as part of NCERT syllabus. The problems here are a happy blend of the straight and the twisted, the simple and the difficult and the easy and the challenging.

DEFINITION

If f and g are functions of x such that $g'(x) = f(x)$, then the function g is called a Primitive Or Antiderivative Or Integral of $f(x)$ w.r.t. x and is written symbolically as $\int f(x)dx = g(x)+c \Leftrightarrow \frac{d}{dx}\{g(x)+c\} = f(x)$, where c is called the **constant of integration.**

RESULTS

- $\int \text{cosec}^2(ax+b)dx = -\frac{1}{a}\cot(ax+b)+c$
- $\int \sec(ax+b)\cdot\tan(ax+b)dx = \frac{1}{a}\sec(ax+b)+c$
- $\int \text{cosec}(ax+b)\cdot\cot(ax+b)dx = -\frac{1}{a}\text{cosec}(ax+b)+c$
- $\int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b)+c$
- $\int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b)+c$
- $\int \tan(ax+b)dx = \frac{1}{a}\ln\sec(ax+b)+c$
- $\int \sec^2(ax+b)dx = \frac{1}{a}\tan(ax+b)+c$
- $\int \sec x\,dx = \ln(\sec x+\tan x)+c$
  or $\ln\tan\left(\frac{x}{4}+\frac{x}{2}\right)+c$
- $\int \text{cosec}\,x\,dx = \ln(\text{cosec}x-\cot x)+c$
  or $\ln\tan\frac{x}{2}+c$ or $-\ln(\text{cosec}x+\cot x)$
- $\int \cot(ax+b)\,dx = \frac{1}{a}\ln\sin(ax+b)+c$
- $\int \frac{dx}{ax+b} = \frac{1}{a}\ln(ax+b)+c$
- $\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)}+c \quad n\neq -1$
- $\int e^{ax+b}dx = \frac{1}{a}e^{ax+b}+c$
- $\int a^{px+q}dx = \frac{1}{p}\frac{a^{px+q}}{\ln a}(a>0)+c$
- $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left[x+\sqrt{x^2+a^2}\right]+c$
- $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left[x+\sqrt{x^2-a^2}\right]+c$
- $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}+c$
- $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\frac{a+x}{a-x}+c$
- $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\frac{x-a}{x+a}+c$
- $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}+c$

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He trains IIT and Olympiad aspirants.

- $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a}+c$
- $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log|x\sqrt{x^2-a^2}|+c$
- $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}+c$
- $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log|x+\sqrt{x^2+a^2}|$
- $\int e^{ax}\cdot\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)+c$
- $\int e^{ax}\cdot\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)+c$

TYPES OF INTEGRATION

- **Substitution** or change of independent variable.

  Integral $I = \int f(x)\,dx$ is changed to

  $\int f(\phi(t))\,f'(t)\,dt$, by a suitable substitution $x = \phi(t)$.
- **Integration by part**

$$\int u\cdot v\,dx = u\int v\,dx - \int\left[\frac{du}{dx}\cdot\int v\,dx\right]dx$$

where u and v differentiable function.

Note : While using integration by parts, choose u and v such that

(a) $\int v\,dx$ is simple and

(b) $\int\left[\frac{du}{dx}\cdot\int v\,dx\right]dx$ is simple to integrate.

This is generally obtained, by keeping the order of u and v as per the order of the letters in **ILATE**, where; I-Inverse function, L-Logarithmic function, A-Algebraic function, T-Trigonometric function and E-Exponential function

- **Partial fraction**, spiliting a bigger fraction into smaller fraction by known methods.

INTEGRALS OF THE TYPE

- $\int e^x[f(x)+f'(x)]dx = e^x\cdot f(x)+c$
- $\int [f(x)+xf'(x)]\,dx = xf(x)+c$
- $\int \frac{dx}{x(x^n+1)}\ n\in N$

  Take x^n common and put $1 + x^{-n} = t$.
- $\int [f(x)]^n f'(x)dx$

  or $\int \frac{f'(x)}{[f(x)]^n}dx$ put $f(x)=t$ and proceed.
- $\int \frac{dx}{ax^2+bx+c}, \int \frac{dx}{\sqrt{ax^2+bx+c}}, \int \sqrt{ax^2+bx+c}\,dx$

  Express $ax^2 + bx + c$ in the form of perfect square & then apply the standard results .
- $\int \frac{px+q}{ax^2+bx+c}dx, \int \frac{px+q}{\sqrt{ax^2+bx+c}}dx$

  Express $px + q = A$ (differential co-efficient of denominator) + B.
- $\int \frac{dx}{x^2\,(x^n+1)^{(n-1)/n}}\ n\in N,$

  take x^n common and put $1 + x^{-n} = t^n$.
- $\int \frac{dx}{a+b\sin^2 x}$ or $\int \frac{dx}{a+b\cos^2 x}$

  or $\int \frac{dx}{a+\sin^2 x + b\sin x\cos x + c\cos^2 x}$

  Multiply N^r and D^r by $\sec^2 x$ and put $\tan x = t$.
- $\int \frac{dx}{a+b\sin x}$ or $\int \frac{dx}{a+b\cos x}$

  or $\int \frac{dx}{a+b\sin x + c\cos x}$

  Convert sines and cosines into their respective tangents of half the angles , put $\tan\frac{x}{2}=t$
- $\int \frac{a\cdot\cos x + b\cdot\sin x + c}{l\cdot\cos x + m\cdot\sin x + n}dx.$

  Express $Nr \equiv A(Dr) + B\,\frac{d}{dx}\,(Dr) + c$ and proceed .
- $\int \frac{x^2+1}{x^4+Kx^2+1}dx$ or $\int \frac{x^2-1}{x^4+Kx^2+1}dx$

  where K is any constant.

  Divide Nr & Dr by x^2 and proceed.

- $\int \frac{dx}{(ax+b)\sqrt{px+q}}$

  and $\int \frac{dx}{(ax^2+bx+c)\sqrt{px+q}}$;

  put $px + q = t^2$.

- $\int \frac{dx}{(ax+b)\sqrt{px^2+qx+r}}$, put $ax+b=\frac{1}{t}$;

  $\int \frac{dx}{(ax^2+bx+c)\sqrt{px^2+qx+r}}$, put $x=\frac{1}{t}$

- $\int \sqrt{\frac{x-\alpha}{\beta-x}}\,dx$ or $\int \sqrt{(x-\alpha)(\beta-x)}dx$;

  put $x = \alpha\cos^2\theta + \beta\sin^2\theta$

  $\int \sqrt{\frac{x-\alpha}{x-\beta}}\,dx$ or $\int \sqrt{(x-\alpha)(x-\beta)}$;

  put $x = \alpha\sec^2\theta - \beta\tan^2\theta$

  $\int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}$; put $x-\alpha=t^2$ or $x-\beta=t^2$

DEFINITE INTEGRAL

$$\int_a^b f(x)dx = [F(x)]_a^b F(b) - F(a)$$

PROPERTIES OF DEFINITE INTEGRAL

- $\int_a^b f(x)dx = \int_a^b f(t)dt$ provided f is same
- $\int_a^b f(x)dx = -\int_b^a f(x)\,dx$
- $\int_a^b f(x)dx = \int_a^c f(x)\,dx + \int_c^b f(x)dx,$
- $\int_{-a}^{a} f(x)dx = 0$, if $f(x)$ is an odd function

  i.e. $f(x) = -f(-x)$.

  $= 2\int_0^a f(x)dx$, if $f(x)$ is an even function

  i.e. $f(x) = f(-x)$

- $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx,$
- $\int_0^a f(x)dx = \int_0^a f(a-x)dx$
- $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$
- $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$, if $f(2a-x) = f(x)$ &

  $= 0$, if $f(2a-x) = -f(x)$

- $\int_0^{na} f(x)\,dx = n\int_0^a f(x)dx$; where '$a$' is the period

  of the function i.e. $f(a + x) = f(x)$

- $\int_{a+nT}^{b+nT} f(x)\,dx = \int_a^b f(x)dx$

  where $f(x)$ is periodic with period T & $n \in I$.

- $\int_{ma}^{na} f(x)\,dx = (n-m)\int_0^a f(x)dx$

  if $f(x)$ is periodic with period 'a'.

- If $f(x) \le \phi(x)$ for $a \le x \le b$, then

  $\int_a^b f(x)\,dx \le \int_a^b \phi(x)\,dx$

- $\left|\int_a^b f(x)dx\right| \le \int_a^b |f(x)|\,dx$
- If $f(x) \ge 0$ on the interval $[a, b]$, then

  $\int_a^b f(x)\,dx \ge 0$

WALLI'S FORMULA

$$\int_0^{\pi/2} \sin^n x \cdot \cos^m x\,dx$$

$$= \frac{[(n-1)(n-3)(n-5)\ldots 1 \text{ or } 2]\,[(m-1)(m-3)\ldots 1 \text{ or } 2]}{(m+n)(m+n-2)(m+n-4)\ldots 1 \text{ or } 2}K$$

Where $K = \frac{\pi}{2}$ if both m and n are even $(m, n \in N)$;

$= 1$ otherwise

DERIVATIVE OF ANTIDERIVATIVE FUNCTION

If $h(x)$ and $g(x)$ are differentiable functions of x then,

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = f[h(x)]\cdot h'(x) - f[g(x)]\cdot g'(x)$$

DEFINITE INTEGRAL AS LIMIT OF A SUM

$$\int_a^b f(x)dx = \lim_{n\to\infty}\frac{1}{n}[f(a)+f(a+h)+f(a+2h)$$
$$+ \ldots\ldots\ldots\ldots + f(a+\overline{n-1}\,h)]$$
$$= \lim_{h\to 0} h\sum_{r=0}^{n-1} f(a+rh) \text{ where } b-a=nh$$

If $a = 0$ and $b = 1$ then,

$$\lim_{n\to\infty} h\sum_{r=0}^{n-1} f(rh) = \int_0^1 f(x)dx; \text{ where } nh=1$$

or $$\lim_{n\to\infty}\left(\frac{1}{n}\right)\sum_{r=1}^{n-1} f\left(\frac{r}{n}\right) = \int_0^1 f(x)\,dx$$

ESTIMATION OF DEFINITE INTEGRAL

- For a monotonic increasing function in (a, b);

$$f(a)\cdot(b-a) < \int_a^b f(x)dx < f(b).(b-a)$$

- For a monotonic decreasing function in (a, b);

$$f(b)\cdot(b-a) < \int_a^b f(x)dx < f(a)\cdot(b-a)$$

SOME IMPORTANT SUMS

- $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots\ldots\infty=\frac{\pi^2}{8}$
- $\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+\frac{1}{8^2}+\ldots\ldots\infty=\frac{\pi^2}{24}$
- $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\ldots\infty=\frac{\pi^2}{12}$
- $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}+\ldots\ldots\infty=\ln 2$
- $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots\ldots\infty=\frac{\pi^2}{6}$

PROBLEMS
SECTION-I
Single Correct Answer Type

1. $\int \frac{\sqrt{1-x^2}-x}{\sqrt{1-x^2}\left(1+x\sqrt{1-x^2}\right)}dx$ is

(a) $2\tan^{-1}\left(x+\sqrt{1-x^2}\right)+c$

(b) $\tan^{-1}\left(x+\sqrt{1-x^2}\right)+c$

(c) $2\tan^{-1}\left(x-\sqrt{1-x^2}\right)+c$

(d) $2\cot^{-1}\left(x+\sqrt{1-x^2}\right)+c$

2. $\int \cos^2 x\,dx =$

(a) $\frac{x}{2}+\frac{1}{4}\sin 2x+c$ (b) $\frac{x}{2}-\frac{1}{4}\sin 2x+c$

(c) $-2\cos x\sin x + c$ (d) None

3. $\int \frac{3\,dx}{2x^2-x-1} =$

(a) $\log\left|\frac{x-1}{x+1}\right|+c$ (b) $\log\left|\frac{x+1}{2x+1}\right|+c$

(c) $\log\left|\frac{x-1}{2x-1}\right|+c$ (d) $\log\left|\frac{x-1}{2x+1}\right|+c$

4. If $\int \frac{dx}{\sqrt{x}(x+9)} = f(x)$ + constant, then $f(x) =$

(a) $\frac{2}{3}\tan^{-1}\sqrt{x}$ (b) $\frac{2}{3}\tan^{-1}\left(\frac{\sqrt{x}}{3}\right)$

(c) $\tan^{-1}\sqrt{x}$ (d) $\sqrt{x}\tan^{-1}\left(\frac{\sqrt{x}}{3}\right)$

5. $\int x^2 e^x\,dx =$

(a) $e^x(x^2-1)+1$ (b) $e^x(x^2+2x+1)$

(c) $e^x(x^2-2x+2)+c$ (d) None of these

6. $\int |x|\,dx =$

(a) $\frac{x^2}{2}+c$ (b) $\frac{x|x|}{2}+c$

(c) x^2+c (d) $\frac{|x|^2}{2}+c$

7. If $\int \sqrt{\tan x}\, dx = a\tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x}}\right) + b\, ln\left(\frac{\tan x + 1 + \sqrt{2\tan x}}{\tan x + 1 - \sqrt{2\tan x}}\right) + C,$

then $a + b =$

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{2\sqrt{2}}$ (d) $\frac{1}{4}$

8. A point P moves in xy-plane in such a way that $[|x|] + [|y|] = 1$, where $[\cdot]$ denotes the greatest integer function. Area of the region representing all possible positions of the point P is equal to

(a) 4 sq. unit (b) 16 sq. unit

(c) $2\sqrt{2}$ sq. unit (d) 8 sq. unit

9. If $\int \frac{dx}{x^r(1+x^r)^{1/r}}, r \in N = \alpha(1+x^{-3})^{\beta} + c,$ then the quadratic equation having roots α and β, is

(a) $6x^2 - x - 2 = 0$ (b) $6x^2 - x + 2 = 0$

(c) $6x^2 + x - 2 = 0$ (d) $6x^2 + x + 2 = 0$

10. The integral

$\int \sin 2x\left(1 - \frac{3}{2}\cos x\right) e^{\sin^2 x + \cos^3 x} dx$

is equal to _____

(a) $e^{(\sin^2 x + \cos^{3/2} x)^2} + c$ (b) $-\frac{e^{(\sin^2 x + \cos^{3/2} x)}}{2} + c$

(c) $e^{+(\sin^2 x + \cos^3 x)} + c$ (d) $-e^{-(\sin^2 x + \cos^3 x)} + c$

11. If $\int \frac{\cos x - \sin x}{1 + \sin 2x} dx = f(x) + c,$ where c is an arbitrary constant and $f(0) = -1$, then $f\left(\frac{\pi}{4}\right)$ equals

(a) $-\frac{1}{\sqrt{2}}$ (b) $-\sqrt{2}$ (c) 2 (d) $3\sqrt{2}$

12. $\int\left(\sqrt{\frac{\cos x}{x}} - \sqrt{\frac{x}{\cos x}}.\sin x\right) dx$ equals

(a) $-\sqrt{x\cos x} + C$ (b) $\sqrt{x\sin x} + C$

(c) $2\sqrt{x\cos x} + C$ (d) $C - 2\sqrt{x\cos x}$

13. $\int \frac{x^6(x+1)}{\sqrt{5x^{10} + 6x^9 + x^4}} dx$ is equal to

(a) $\frac{\sqrt{5x^6 + 6x^5 + 1}}{15} + c$

(b) $\frac{\sqrt{5x^8 + 6x^7 + x^2}}{30} + c$

(c) $\frac{\sqrt{5x^8 + 6x^7 + x^2}}{15} + c$

(d) $(5x^{10} + 6x^5 + x^4)^{1/2} + c$

14. If $I = \int \tan x \tan 2x \tan 3x\, dx,$ then I equals

(a) $\log|\cos x| + (1/2)\log|\cos 2x| + (1/3)\log|\cos 3x| + C$

(b) $\log|\cos x| - (1/2)\log|\cos 2x| - (1/3)\log|\cos 3x| + C$

(c) $\log|\cos x| + (1/2)\log|\cos 2x| - (1/3)\log|\cos 3x| + C$

(d) none of these

15. A function $g(x)$ is given by

$g(x) = \frac{x}{1 + (\log_e x)(\log_e x)....}$

where $x \in [1, \infty)$, then $\int_1^{2e} f(x)dx$ is equal to

(a) $\frac{1}{2}\left(e - \frac{1}{e}\right)$ (b) $\frac{1}{2}\left(e + \frac{1}{e}\right)$

(c) $\frac{1}{2}(e^2 - 1)$ (d) $\frac{1}{2}(e^2 + 1)$

16. If $f(x) = \begin{vmatrix} x & \cos x & e^{x^2} \\ \sin x & x^2 & \sec x \\ \tan x & 1 & 2 \end{vmatrix},$ then the value of $\int_{-\pi/2}^{\pi/2} f(x)dx$ is equal to

(a) 5 (b) 3 (c) 1 (d) 0

17. $\lim_{x\to 0} \frac{1}{x}\left(\int_y^c e^{\sin^2 t} dt - \int_{x+y}^c e^{\sin^2 t} dt\right)$ is equal to (where c is a constant)

(a) $e^{\sin^2 y}$ (b) $\sin 2y\, e^{\sin^2 y}$

(c) 0 (d) none of these

18. If $\int_1^x \frac{dt}{1 + [\log_{10} t]} = 2005,$ then $x =$ _____ where $[\cdot]$ is g.i.f

(a) 7604 (b) 7408 (c) 5010 (d) 5200

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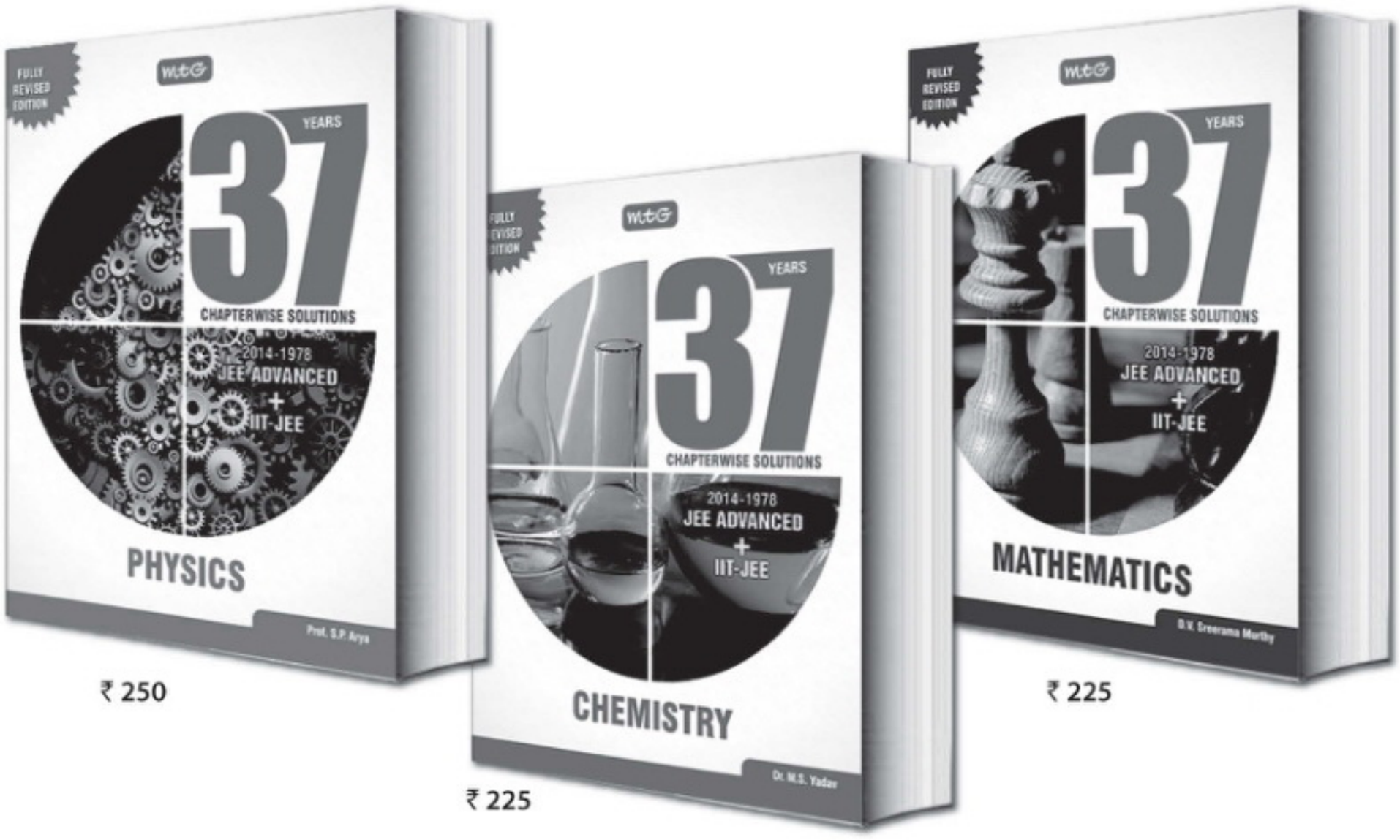

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19. In the given coordinate plane, the region M consists of all the points (x, y) satisfying the inequalities $y \le x, y \ge 0$ and $y \le 2 - x$ simultaneously. The region N which varies with parameter 't', consists all the points (x, y) satisfying $t \le x \le t + 1$ and $0 \le t \le 1$ and simultaneously area of $M \cap N$ is

(a) $-t^2 + t + \frac{1}{2}$ (b) $-2t^2 + 2t$

(c) $1 - \frac{t^2}{2}$ (d) $\frac{(t-1)^2}{2}$

20. The value of the definite integral $\int_{-2008}^{2008} \frac{f'(x) + f'(-x)}{(2008)^x + 1} dx$ equals

(a) $f(2008) + f(-2008)$ (b) $f(2008) - f(-2008)$

(c) 0 (d) $f(-2008) - f(2008)$

21. The value of $S = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{m+n+mn}{2^m(2^m + 2^n)}$ is equal to

(a) 0 (b) 12 (c) 9 (d) 6

22. $\int_0^{\pi/2} \frac{dx}{1 + \tan^{13} x}$ is

(a) 0 (b) 1 (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

23. The value of $\int_{e^{-1}}^{e^2} \left|\frac{\log x}{x}\right| dx$ is

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) 3 (d) 2

24. If A is the area between the curve $y = x^6(1 - x)^7$ and x-axis, then the number of prime factors of A^{-1} is ____

(a) 3 (b) 4 (c) 2 (d) 5

25. $\int_0^{\pi} \frac{dx}{(5 + 4\cos x)^2} =$

(a) $\frac{3\pi}{7}$ (b) $\frac{5\pi}{17}$ (c) $\frac{5\pi}{27}$ (d) $\frac{5\pi}{26}$

26. $\int_{-\pi}^{\pi} (1 - x^3)\sin^2 x \cos^4 x dx$

(a) 0 (b) $\frac{\pi}{16}$ (c) $\frac{\pi}{8}$ (d) $\frac{\pi^2}{8}$

SECTION-II

More than One Correct Answer Type

27. Let $f(x) = \int_0^x \frac{e^t}{t} dt\ (x > 0)$; then $e^{-a}[f(x + a) - f(1 + a)] =$ .....

(a) $\int_0^x \frac{e^t}{(t+a)} dt$ (b) $\int_1^x \frac{e^t}{t+a} dt$

(c) $e^{-a} \int_{1+a}^{x+a} \frac{e^t}{t} dt$ (d) $\int_0^x \frac{e^{t-a}}{(t+a)} dt$

28. A function 'f', continuous on the positive real axis, has the property that for all choices of $x > 0$ and $y > 0$, the integral $\int_x^{xy} f(t)\, dt$ is independent of 'x', (and therefore depends only on y). If $f(2) = 2$, then $\int_1^e f(t)\, dt$ is equal to

(a) e (b) $4e$

(c) 4 (d) none of these

29. If $x > 0$ and $\int_0^x [x] dx = [x]\left(\frac{1}{2}A + B\right)$, where [.] denotes the greatest integer function, then

(a) $A = [x] - 1$ (b) $B = x - [x]$

(c) $A = [x] + 1$ (d) $B = x + [x]$

30. If $\alpha, \beta\ (\alpha < \beta)$ are 2 roots of $(6x + 1)x = 1 + \left[\cos\frac{\pi}{4}\right]$ then $\int_0^{2\alpha} \sin\left(\frac{\pi[x]}{2}\right) dx + \int_0^{3\beta} \cos(\pi[x]) dx =$ ____. Where denotes greatest integer function

(a) $\alpha + \beta$ (b) 2

(c) 0 (d) $[2\alpha + 9\beta]$

31. Let $I = \int_{k\pi}^{(k+1)\pi} \frac{|\sin 2x| dx}{|\sin x| + |\cos x|}, (k \in N)$ and $J = \int_0^{\frac{\pi}{4}} \frac{dx}{\sin x + \cos x}$, then which of the following holds good?

(a) $I = 2\int_0^{\pi/2} \frac{\sin 2x\,dx}{\sin x + \cos x}$

(b) $I = 4 - 4J$

(c) $I = 4 - 2J$ (d) $I = 2 - 2J$

32. $I = \int e^{x\sin x+\cos x}\left(\frac{x^4\cos^3 x - x\sin x + \cos x}{x^2\cos^2 x}\right)dx,$

then I equals

(a) $e^{x\sin x+\cos x}\left(x - \frac{\sec x}{x}\right)+C$

(b) $e^{x\sin x} + \cos x\left(x\sin x - \frac{\cos x}{x}\right)+C$

(c) $e^{x\sin x} + \cos x\left(\frac{x}{\tan x} - \frac{\sec x}{x}\right)+C$

(d) $xe^{x\sin x+\cos x} - e^{x\sin x+\cos x}\frac{1}{x\cos x} - C$

33. If

$\int xe^{-5x^2}\sin 4x^2 dx = Ke^{-5x^2}(A\sin 4x^2 + B\cos 4x^2)+C,$

then

(a) $K = -1/82$ (b) $K = 1/82$

(c) $A = 5$ (d) $B = 4$

34. If $\int \frac{xe^x}{\sqrt{1+e^x}}dx = f(x)\sqrt{1+e^x} - 2\log g(x)+C,$ then

(a) $f(x) = x - 1$ (b) $g(x) = \frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}$

(c) $g(x) = \frac{\sqrt{1+e^x}+1}{\sqrt{1+e^x}-1}$ (d) $f(x) = 2(x - 2)$

35. Let $f(x)$ be a function such that $f''(x) = f(x)$ and $f'(0) = f(0) = 0$. The value of $\int \frac{f(x)}{4-f(x)}dx$ is equal to

(a) $\log_e(4 - f(x)) + c$ (b) $-\log_e(4 - f(x)) + c$

(c) $\log_e\left(\frac{1+f(x)}{4-f(x)}\right)+c$

(d) $-\log_e\left(\frac{1+f(x)}{4-f(x)}\right)+c$

36. If $A(x+y) = A(x)A(y)\ \forall x,\ y$ and $A(0) \neq 0$ and $B(x) = \frac{A(x)}{1+(A(x))^2}$, then

(a) $\int_{-2010}^{2010} B(x)dx = \int_0^{2011} B(x)dx$

(b) $\int_{-2010}^{2011} B(x)dx - \int_0^{2010} B(x)dx = \int_0^{2011} B(x)dx$

(c) $\int_{-2010}^{2011} B(x)dx = 0$

(d) $\int_{-2010}^{2010} (2B(-x) - B(x))dx = 2\int_0^{2010} B(x)dx$

37. The value of $\int_3^4 \frac{1}{\sqrt[3]{\log x}}dx$ is

(a) less than one (b) greater than 1/2

(c) less than two (d) greater than 1/3

38. If $u = \int e^{ax}\cos bx\ dx$ and $v = \int e^{ax}\sin bx\ dx$, then

(a) $(a^2 + b^2)(u^2 + v^2) = e^{2ax}$

(b) $(a^2 - b^2)(u^2 - v^2) = e^{2ax}$

(c) $\tan^{-1}\frac{v}{u} + \tan^{-1}\frac{b}{a} = bx$

(d) $\tan^{-1}\frac{v}{u} - \tan^{-1}\frac{b}{a} = ax$

39. If $4 + \left|\int_o^t (ax+b)dx\right| = t(4-t)$, then locus of (a, b) is

(a) a straight line whose inclination is $3\pi/4$

(b) a set of parallel lines

(c) a set of straight lines whose inclination is $\tan^{-1}2$

(d) Perpendicular distance from origin to the locus is 0

40. If A_1 is the area bounded by $y = \log_e|x|$ and $y = 0$, A_2 is the area bounded by $y = |\log_e|x||$ and $y = 0$, A_3 is the area bounded by $y = \log_e|x||\log_e|x||$ and $y = 0$ then

(a) $A_1 + A_2 + A_3 = 8$ (b) $A_1 - A_2 = 0$

(c) $\frac{2A_1 + 4A_2 + A_3}{4} = 4$ (d) $5A_2 - 3A_1 + A_3 = 8$

41. Suppose $I_1 = \int_0^{\frac{\pi}{2}} \cos(\pi\sin^2 x)dx,$

$I_2 = \int_0^{\frac{\pi}{2}} \cos(2\pi \sin^2 x)dx$ and $I_3 = \int_0^{\frac{\pi}{2}} \cos(\pi \sin x)dx$ then

(a) $I_1 = 0$ (b) $I_2 + I_3 = 0$

(c) $I_1 + I_2 + I_3 = 0$ (d) $I_2 = I_3$

SECTION-III

Comprehension Type

Paragraph for Question No. 42 and 43

If $f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t\cos x) f(t)\,dt$, then

42. The range of $f(x)$ is

(a) $\left[\frac{-\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right]$ (b) $\left[\frac{-\sqrt{5}}{3}, \frac{\sqrt{5}}{3}\right]$

(c) $\left[\frac{-\sqrt{5}}{2}, \frac{\sqrt{5}}{2}\right]$ (d) $\left[\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right]$

43. The value of $\int_0^{\pi/2} f(x)\,dx$ is

(a) 1 (b) –2 (c) –1 (d) 2

Paragraph for Question No. 44 to 46

Sometimes instead of evaluating an integral we need to express the value of one integral in terms of the other. Define

$$I = \int_0^{\pi} \frac{\cos x}{(x+2)^2} dx, J = \int_0^{1} \frac{e^x}{1+x} dx$$

44. $\int_0^{\pi/2} \frac{\sin x \cos x}{x+1} dx$ in terms of I equals

(a) $\frac{1}{4} - \frac{1}{2(\pi+2)} + \frac{1}{2}I$ (b) $\frac{1}{4} - \frac{1}{\pi+2} + \frac{1}{2}I$

(c) $\frac{1}{4} + \frac{1}{\pi+2} - \frac{1}{2}I$ (d) $\frac{1}{4} + \frac{1}{2(\pi+2)} - \frac{1}{2}I$

45. $\int_3^4 \frac{e^{-x}}{x-5} dx$ in terms of J equals

(a) Je^{-4} (b) $-Je^{-4}$ (c) Je^4 (d) $-Je^4$

46. $\int_0^1 \frac{x^2}{(x+1)^2} e^x dx$ in terms of J equals

(a) $\frac{e}{2} - J$ (b) $J - \frac{e}{2}$

(c) $\frac{e}{2} + 1 - J$ (d) $J - 1 + \frac{e}{2}$

SECTION-IV

Matrix Match Type

47.

Column-I		Column-II	
(A)	The area enclosed by $\|x\| + \|y\| = 1$ is	(p)	6
(B)	The area bounded by $y = \|x - 1\|$ and $y = 3 - \|x\|$ is	(q)	2
(C)	Area bounded by $y = x + 2$; $y = 2 - x$; and $x = 2$ is	(r)	8
(D)	The area bounded by $y = 1 + \frac{8}{x^2}$ with x-axis and ordinates $x = 2$ and $x = 4$ is	(s)	4

48.

Column-I		Column-II	
(A)	$\int_0^{\frac{\pi}{2}} \left(\frac{x}{\sin x}\right)^2 dx$	(p)	$2\log\|\sec x\| + k$
(B)	$\int \sin 2x.d(\tan x)$	(q)	$\pi + 2$
(C)	$\int_0^2 \sqrt{\frac{2+x}{2-x}}\, dx$	(r)	$\pi \log 2$
(D)	$2\int_0^1 \frac{\sin^{-1} x}{x} dx$	(s)	$\frac{-\log 2x}{2} + k$

49.

Column-I		Column-II	
(A)	If $I_1 = \int_0^1 x^4(1-x)^5 dx$, $I_2 = \int_0^1 x^5(1-x)^5 dx$ then $\frac{I_2}{I_1} =$	(p)	$\frac{1}{2}$
(B)	If $I_1 = \int_0^{\pi} \cos^4 x \sin^6 x dx$, $I_2 = \int_0^{2\pi} \cos^4 x \sin^4 x dx$ then $\frac{I_1}{I_2} =$	(q)	$\frac{1}{3}$

(C)	If $f(x)=\dfrac{\sin^2 x}{(1-e^{-3x})\sin^2 2x}$ then $\int\limits_{\frac{-\pi}{4}}^{\frac{\pi}{4}} f(x)dx=$	(r)	$\frac{1}{4}$
(D)	$\int\limits_0^{\frac{\pi}{2}}(\sqrt{\sin x}+\sqrt{\cos x})^{-4}dx$	(s)	$\frac{5}{11}$

SECTION-V

Integer Answer Type

50. If the area between the curves $y = x - x^2$ and $y = mx$ is $\frac{9}{2}$ then the sum of all values of m is__

51. If $I=-\int\limits_0^1 (x+1)e^x ln(x)dx$, then the least integer greater than I is ___

52. If $f(x) = e^{\cos x}\sin x$, $|x| \le 2$

$= 2$, otherwise then $\int\limits_{-2}^{3} f(x)dx =$

53. Let $f(x)$ be a function defined by

$$f(x)=\int\limits_1^x t(t^2-3t+2)dt, 1\le x\le 3$$

then number of positive integers in the range of $f(x)$ is

54. Let $f(x)$ be a polynomial satistying $f(0) = 2$, $f(0) = 3$, $f'(x) = f(x)$, $f(4) = \dfrac{k(e^8-1)}{e^4}$ then $[k]$ is ___(where $[x]$ is step function)

55.

$$\int\frac{x^2dx}{(x\sin x+\cos x)^2}=\frac{\sin^l x - x\cos^p x}{(x\sin x+\cos x)}+c=l+p=_____$$

56. The minimum area of circle which touches the parabolas $y = x^2 + 1$ and $y^2 = x - 1$ is $\frac{\lambda\pi}{32}$ square units then λ = _____

57. If $\int\dfrac{dx}{(\sec x+\operatorname{cosec} x+\tan x+\cot x)^2}=$

$$\frac{x}{a}+\frac{\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)}{b}+\frac{\cos 2x}{c}+d$$

then $|a + b + c|$ is

58. The area enclosed by the curve $xy^2 = 4(2 - x)$ and y-axis is λ sq. unit, then the value of $\cos(1824\lambda)$ must be

59. If $f(x)=\int\limits_{\pi^2/16}^{x^2}\dfrac{\sin x\cdot\sin\sqrt{\theta}}{1+\cos^2\sqrt{\theta}}\cdot d\theta$ then. If the value of $f'\left(\frac{\pi}{2}\right)=\lambda$, then $[\lambda]$ = ___where [.] g.i.f

60. Let $u=\int\limits_0^1\frac{\ln(x+1)}{x^2+1}dx$ and $v=\int\limits_0^{\pi/2}\ln(\sin 2x)dx$ then $4u + v$ = _____

SOLUTIONS

1. (a) : Let $x = \sin\theta$; $dx = \cos\theta d\theta$

$$\int\frac{(\cos\theta-\sin\theta)\cos\theta\, d\theta}{\cos\theta(1+\sin\theta\cos\theta)}$$

2. (a) : $\int\cos^2 xdx=\int\frac{1+\cos 2x}{2}dx$

$$=\frac{1}{2}x+\frac{1}{2}\frac{\sin 2x}{2}c$$

3. (d) : $\int\frac{3\,dx}{(2x+1)(x-1)}=-\int\left(\frac{2}{2x+1}-\frac{1}{x-1}\right)dx$

$$=-\log|2x+1|+\log|x-1|+c$$

$$=\log\left|\frac{x-1}{2x+1}\right|+c$$

4. (b) : Put $x = t^2 \Rightarrow dx = 2t\,dt$

$$\therefore\ \int\frac{dx}{\sqrt{x}(x+9)}=\int\frac{2t\,dt}{t\cdot(t^2+3^2)}$$

$$=2\int\frac{1}{t^2+(3)^2}dt=\frac{2}{3}\tan^{-1}\left(\frac{t}{3}\right)+c$$

$$=\frac{2}{3}\tan^{-1}\left(\frac{\sqrt{x}}{3}\right)+c$$

5. (c) : $\int x^2 e^x\,dx=e^x(x^2-2x+2)+c$

($\therefore$ by integrate by parts)

6. (b) : $\int |x|\,dx = \frac{x|x|}{2} + c$

7. (c) : $\tan x = t^2 \Rightarrow \sec^2 x\,dx = 2t\,dt$

$$I = 2\int \frac{t^2\,dt}{t^4+1} = \int \frac{t^2+1+t^2-1}{t^4+1}\,dt$$

$$= \int \frac{t^2+1}{t^4+1}\,dt + \int \frac{t^2-1}{t^4+1}\,dt$$

$$= \int \frac{1+\frac{1}{t^2}}{\left(t-\frac{1}{t}\right)^2+2}\,dt + \int \frac{1-\frac{1}{t^2}}{\left(t+\frac{1}{t}\right)^2-2}\,dt$$

8. (d) : If $[|x|] = 1$ and $[|y|] = 0$, then $1 \le |x| < 2$, $0 \le |y| < 1 \Rightarrow x \in (-2, -1] \cup [1, 2), y \in (-1, 1)$
If $[|x|] = 0$ and $[|y|] = 1$, then $x \in (-1, 1)$ and $y \in (-2, 1] \cup [1, 2]$
$\therefore$ Required area $= 4(2-1)[1-(-1)] = 8$

9. (a) : $\int \frac{dx}{x^r(1+x^r)^{1/r}} = \frac{1}{(1-r)}(1+x^{-r})^{1-\frac{1}{r}} + c$

on comparing, $r = 3$

$\Rightarrow \alpha = \frac{-1}{2}, \beta = \frac{2}{3}$

$\Rightarrow 6x^2 - x - 2 = 0$

10. (c) : Put $\sin^2 x + \cos^3 x = t$.

11. (a) : Put $\sin x + \cos x = t$ in the integral.

$\therefore I = \int \frac{dt}{t^2} = \frac{-1}{t} + c \Rightarrow f(x) = \frac{-1}{\sin x + \cos x}$

12. (c) : $\int \frac{\cos x - x\sin x}{\sqrt{x\cos x}}\,dx$

Put $x\cos x = 6^2$

$\Rightarrow (\cos x - x\sin^2 x)dx = 2t\,dt$

$$= \int \frac{2t\ dt}{t} = 2t + C = 2\sqrt{x\cos x} + C$$

13. (a) : $I = \int \frac{(x^5+x^4)}{\sqrt{5x^6+6x^5+1}}\,dx = \frac{\sqrt{5x^6+6x^5+1}}{15} + c$

14. (c) : $\tan 3x = \frac{\tan x + \tan 2x}{1 - \tan x \tan 2x}$

$\Rightarrow \tan x \tan 2x \tan 3x = \tan 3x - \tan x - \tan 2x$

$$\int \tan x \tan 2x \tan 3x\,dx$$

$$= \log|\cos x| + \frac{1}{2}\log|\cos 2x| - \frac{1}{3}\log|\cos 3x| + c$$

15. (c) : Let $f(x) = (\log_e x)\cdot(\log_e x).....\infty$

$$\left.\begin{array}{l} f(x) \to 0 \text{ when } 1 < x < e \\ f(x) = 1 \quad \text{when } x = e \\ f(x) \to \infty \text{ when } e < x \end{array}\right\} g(x) = \begin{cases} x, & 1 < x < e \\ \frac{x}{2}, & x = e \\ 0, & e < x < 2e \end{cases}$$

$$Rq = \int_1^e x\,da + \int_e^{2e} 0\,dx = \frac{x^2}{2}\Big]_1^e = \frac{1}{2}(e^2-1)$$

16. (d) : $f(-x) = \begin{vmatrix} -x & \cos x & e^{x^2} \\ -\sin x & x^2 & \sec x \\ -\tan x & 1 & 2 \end{vmatrix} = -f(x)$

$\therefore \int_{-\pi/2}^{\pi/2} f(x)dx = 0$

[$\because$ $f(x)$ is an odd function]

17. (a) : $\lim_{x\to 0} \frac{1}{x}\left(\int_y^c e^{\sin^2 t}dt - \int_{x+y}^c e^{\sin^2 t}dt\right)$

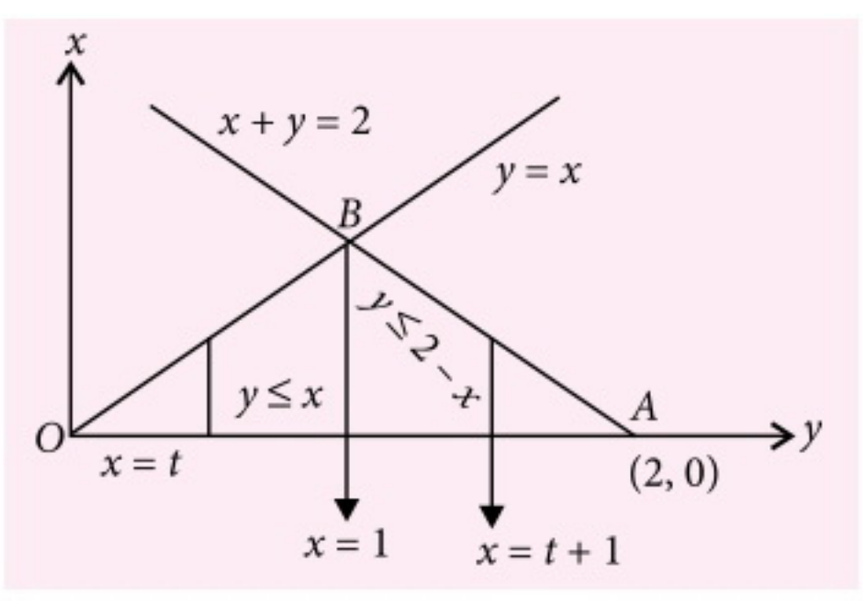
$$= \lim_{x\to 0} \frac{1}{x}\left(\int_y^c e^{\sin^2 t}dt + \int_c^{x+y} e^{\sin^2 t}dt\right)$$

$$= \lim_{x\to 0} \frac{\int_y^{x+y} e^{\sin^2 t}dt}{x} = \lim_{x\to 0} \frac{e^{\sin^2(x+y)} - 0}{1} = e^{\sin^2 y}$$

18. (a) : $\int_1^{10} dt + \int_{10}^{100} \frac{1}{2}dt + \int_{100}^{1000} \frac{dt}{3} + ... + \int_{1000}^{x} \frac{dt}{4} = 2005$

$\Rightarrow x = 7604$

19. (a) : $M = \{(x, y)/y \le x, y \ge 0 \text{ and } y \le 2 - x\}$
$N = \{(x, y)/t \le x \le t + 1 \text{ and } 0 \le t \le 1\}$
if $x \le t + 1, 0 \le t \le 1 \Rightarrow$ Max value of $x = 2$



Area of $M \cap N = \int_t^1 x\,dx + \int_1^{t+1} (2-x)dx$

20. (b) : $I = \int_{-2008}^{2008} \frac{f'(-x)+f'(x)}{(2008)^{-x}+1}dx$

$= \int_{-2008}^{2008} \frac{f'(-x)+f'(x)}{1+(2008)^{x}} \times (2008)^x dx$

$2I = \int_{-2008}^{2008} f'(-x)+f'(x)dx$

$= 2\int_0^{2008} f'(x)+f'(-x)dx$

$I = [f(x)-f(-x)]_0^{2008} = f(2008)-f(-2008)$

21. (d) : Since m, n are identical and independent

$2S = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{m+n+mn}{2^m+2^n}\left[\frac{1}{2^m}+\frac{1}{2^n}\right]$

$= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} (m+n+mn)\frac{1}{2^m}\cdot\frac{1}{2^n}$

$= \sum_{m=0}^{\infty} m.\left(\frac{1}{2}\right)^m \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n + \sum_{m=0}^{\infty}\left(\frac{1}{2}\right)^m \sum_{n=0}^{\infty} n\left(\frac{1}{2}\right)^n$

$+ \sum_{m=0}^{\infty} m\left(\frac{1}{2}\right)^m \sum_{n=0}^{\infty} n\left(\frac{1}{2}\right)^n$

$= 4 + 4 + 2\cdot 2 = 12 \Rightarrow S = 6$

22. (d) : $I = \int_0^{\frac{\pi}{2}} \frac{dx}{1+\tan^{13} x}$ ...(i)

$I_1 = \int_0^{\frac{\pi}{2}} \frac{dx}{1+\tan^{13}\left(\frac{\pi}{2}-x\right)}$;

using $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$= \int_0^{\frac{\pi}{2}} \frac{dx}{1+\cot^{13} x} = \int_0^{\frac{\pi}{2}} \frac{\tan^{13} x}{\tan^{13} x+1}dx$ ...(ii)

$2I = \int_0^{\frac{\pi}{2}} \frac{1+\tan^{13} x}{1+\tan^{13} x}dx$ (Adding (i) &(ii))

$= \int_0^{\frac{\pi}{2}} 1\,dx = \frac{1}{2}[x]_0^{\frac{\pi}{2}} = \frac{\pi}{4}$

23. (b) : The value of $\int_{e^{-1}}^{e^2} \left|\frac{\log x}{x}\right| dx$

$\log x = 0$ when $x = 1$ and hence negative in $\left[\frac{1}{e}, 1\right)$ and positive in $(1, e^2]$

$\therefore \quad I = \int_{\frac{1}{e}}^{1} \left|\frac{\log x}{x}\right| dx + \int_1^{e^2} \left|\frac{\log x}{x}\right| dx$

$= \int_{1/e}^{1} \frac{-\log x}{x}\,dx + \int_1^{e^2} \frac{\log x}{x}\,dx$

$= \left[-\frac{1}{2}(\log x)^2\right]_{\frac{1}{e}}^{1} + \left[\frac{1}{2}(\log x)^2\right]_1^{e^2}$

$= \frac{1}{2}(-1)^2 + \frac{4}{2} - 0 = \frac{5}{2}$

24. (d) : $A = \int_0^1 x^6(1-x)^7 dx$

$= \frac{6!7!}{14!}$ (assuming β – function)

$A^{-1} = \frac{14!}{6!7!} = 2^3\cdot 3\cdot 7\cdot 11\cdot 13$

25. (c) : $\tan\frac{x}{2} = t \Rightarrow I = 2\int_0^{\infty} \frac{1+t^2}{(t^2+9)^2}dt$

But $t = 3\tan\theta$

26. (c) :

$\int_{-\pi}^{\pi} \sin^2 x\cdot\cos^4 x\,dx - \int_{-\pi}^{\pi} \sin^2 x\cdot\cos^4 x\cdot x^3\,dx$

$= 4\int_0^{\pi/2} \sin^2 x\cdot\cos^4 x\,dx - 0$

27. (b, c) : $e^{-a}[f(x+a) - f(1+a)]$

$= e^{-a}\left[\int_0^{x+a} \frac{e^t\cdot dt}{t} - \int_0^{1+a} \frac{e^t}{t}dt\right]$

$$= e^{-a}\left[\int_0^{x+a} \frac{e^t \cdot dt}{t} + \int_{1+a}^{0} \frac{e^t}{t} dt\right]$$

$$= e^{-a}\int_{1+a}^{x+a} \frac{e^t}{t} \cdot dt \ \{\text{Put}, t = y + a, dt = dy\}$$

$$= e^{-a}\int_1^x \frac{e^{y+a}}{y+a} \cdot dy = \int_1^x \frac{e^y \cdot dt}{y+a} = \int_1^x \frac{e^t \cdot dt}{t+a}$$

28. (c) : Since, I is independent of x, $\frac{dI}{dx} = 0$

$\Rightarrow f(xy).y - f(x) = 0$

$\Rightarrow f(x\cdot y)\cdot y = f(x)$

Put, $y = \frac{2}{x}$; $f(2)\left(\frac{2}{x}\right) = f(x)$

$$\Rightarrow \quad f(x) = \frac{4}{x} \Rightarrow \int_1^e \frac{4}{x} \cdot dx = 4\ln e = 4$$

29. (a, b) : Let $[x] = n$

$$I = \int_0^1 [x]dx + \int_1^2 [x]dx + \ldots\ldots + \int_{n-1}^{n} [x]dx + \int_n^x [x]dx$$

$$= [x]\left(\frac{1}{2}([x]-1) + (x - [x])\right)$$

30. (b, d) : $(6x+1)x = 1 + \left[\cos\frac{\pi}{4}\right]$

$$\Rightarrow \quad 6x^2 + x - 1 = 0 \ \Rightarrow \ x = \frac{-1}{2}, \ \frac{1}{3}$$

$\alpha = -\frac{1}{2}$, $\beta = \frac{1}{3}$

$$\therefore \quad G.I = \int_0^{-1} \sin\frac{\pi[x]}{2} dx + \int_0^1 \cos(\pi[x])dx$$

$$= \int_{-1}^{0} 1dx + \int_0^1 1dx = 2$$

Also $[2\alpha + 9\beta] = [-1 + 3] = 2$

31. (a, b) : We have $I = \int_{k\pi}^{(k+1)\pi} \frac{|\sin 2x|dx}{|\sin x| + |\cos x|}$;

Put $x = k\pi + t \ \Rightarrow \ dx = dt$

$$\therefore \quad I = \int_0^{\pi} \frac{|\sin 2x|dx}{|\sin x| + |\cos x|} = 2\int_0^{\pi/2} \frac{\sin 2x\, dx}{\sin x + \cos x}$$

$$= 2\int_0^{\pi/2} \frac{(\sin x + \cos x)^2 - 1}{\sin x + \cos x} dx$$

$$= 2\int_0^{\pi/2} (\sin x + \cos x)dx - 2\int_0^{\pi/2} \frac{dx}{\sin x + \cos x}$$

$$= 4 - 4\int_0^{\pi/4} \frac{dx}{\sin x + \cos x} = 4 - 4J$$

32. (a, d) :

$$I = \int e^{x\sin x + \cos x}\left(x^2\cos x + \frac{\cos x - x\sin x}{(x\cos x)^2}\right)dx$$

$$= \int x(e^{x\sin x+\cos x} \cdot x\cos x)dx$$

$$+ \int e^{x\sin x+\cos x} d\left(-\frac{1}{x\cos x}\right)$$

$$= x \cdot e^{x\sin x+\cos x} - e^{x\sin x+\cos x}\frac{1}{x\cos x} + c$$

33. (a, c, d) : $I = \int e^{-5t}\sin 4t \frac{dt}{2}$ $\qquad (t = x^2)$

$$= \frac{e^{-5t}}{82}(-5\sin 4t - 4\cos 4t) + c$$

$$= -\frac{e^{-5x^2}}{82}(5\sin 4x^2 + 4\cos 4x^2) + c$$

34. (b, d) : $I = x(2\sqrt{1+e^x}) - 2\int\sqrt{1+e^x}dx$

$$= 2x\sqrt{1+e^x} - 2\int \frac{2t^2}{t^2-1}dt \qquad (t = \sqrt{1+e^x})$$

$$= 2x\sqrt{1+e^x} - 4(t + \frac{1}{2}\ln\frac{t-1}{t+1}) + C$$

$$= 2x\sqrt{1+e^x} - 4(\sqrt{1+e^x}) - 2\ln\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1} + C$$

35. (a, b) : $f''(x) = f(x)$

$$\Rightarrow \int f''(x)f'(x)dx$$

$$= \int f'(x)\ f(x)dx \Rightarrow (f'(x))^2 = (f(x))^2$$

$$\Rightarrow \ f'(x) = f(x) \ \text{ or } \ f'(x) = -f(x)$$

$$\Rightarrow \quad \int \frac{f(x)}{4-f(x)} = \int \frac{\pm f'(x)}{4-f(x)} dx$$

$= \log_e(4 - f(x)) + c$ or $-\log_e(4 - f(x)) + c$

36. (b, d) : First note there $B(x)$ is even

$$\int_{-2010}^{2011} B(x)dx = \int_{-2010}^{2010} B(x)dx + \int_{2010}^{2011} B(x)dx$$

$$= 2\int_{0}^{2010} B(x)dx + \int_{2010}^{2011} B(x)dx$$

$$= \int_{0}^{2010} B(x)dx + \int_{0}^{2011} B(x)dx$$

37. (a, b, c) : $\frac{1}{2} < \frac{1}{\sqrt[3]{\ln x}} < 1,2 \ \forall \ x \in [3, 4]$

$$\Rightarrow \int_3^4 \frac{1}{2}\,dx < \int_3^4 \frac{1}{\sqrt[3]{\log x}}dx < \int_3^4 1\,dx \ \text{ or } \int_3^4 2\,dx$$

$$\Rightarrow \frac{1}{2} < \int_3^4 \frac{1}{\sqrt[3]{\log x}}dx < 1 \ \text{ or } \ 2$$

38. (a, c) :

$$u = \int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx) \quad ...(i)$$

$$v = \int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) \quad ...(ii)$$

Squaring and adding (i) and (ii)

$$u^2+v^2 = \frac{e^{2ax}}{a^2+b^2} \Rightarrow (u^2+v^2)(a^2+b^2) = e^{2ax}$$

$$\frac{v}{u} = \frac{a\sin bx - b\cos bx}{a\cos bx + b\sin bx} \Rightarrow \frac{v}{u} = \frac{\tan bx - b/a}{1+\left(\frac{b}{a}\right)\tan bx}$$

$$\frac{v}{u} = \tan(bx - \tan^{-1} b/a)$$

$$\tan^{-1}\frac{v}{u} = bx - \tan^{-1} b/a \Rightarrow \tan^{-1}\frac{v}{u} + \tan^{-1} b/a = bx$$

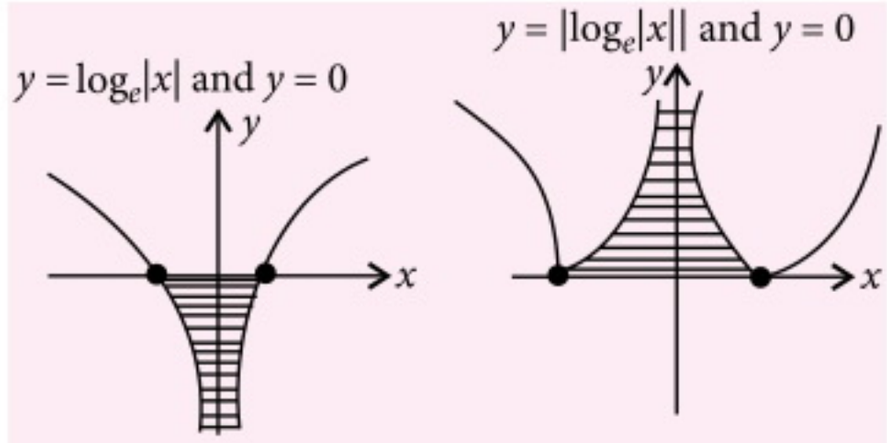
39. (a, d) : $\left|\int_0^t (ax+b)dx\right| + (t-2)^2 = 0$

$$\left|\int_0^t (ax+b)dx\right| = 0 \ \& \ (t-2)^2 = 0$$

$$\left|\int_0^t (ax+b)dx\right| = 0 \Rightarrow a+b=0 \ \ \& \ \ t=2$$

$\therefore$ Locus is $x + y = 0$

40. (a, b, c, d) :



$A_1 = 1 + 1 = 2$ sq. units $\quad A_2 = 1 + 1 = 2$ sq. units

$A_3 = 2 + 2 = 4$ sq. units

41. (a, b, c, d) :

$$I_1 = \int_0^{\frac{\pi}{2}} \cos(\pi\sin^2 x)dx = \int_0^{\frac{\pi}{2}} \cos(\pi\cos^2 x)dx$$

$$\Rightarrow 2I_1 = \int_0^{\frac{\pi}{2}} 2\cos\left(\frac{\pi}{2}\right)\cos\frac{(\pi(\sin^2 x - \cos^2 x))}{2}dx = 0$$

$\Rightarrow I_1 = 0$

$$I_2 = \int_0^{\pi/2} \cos(2\pi\sin^2 x)\,dx = \int_0^{\pi/2} \cos\,(2\pi\cos^2 x)dx$$

$$\Rightarrow \ 2I_2 = \int_0^{\pi/2} 2\cos\pi\cos(-\pi\cos 2x)dx$$

$$\Rightarrow \ I_2 = -\int_0^{\pi/2} \cos(\pi\cos 2x)dx$$

Put $2x = t \Rightarrow \ dx = \frac{1}{2}dt$

$$\text{Then } \ I_2 = -\frac{1}{2}\int_0^{\pi} \cos(\pi\cos t)\,dt$$

$$= -\frac{1}{2}\times 2\int_0^{\pi/2} \cos\,(\pi\cos t)dt \ = -\int_0^{\pi/2} \cos\,(\pi\sin t)dt$$

$= -I_3$

$\Rightarrow \ I_2 + I_3 = 0$

Soln.: 42. (b) **43. (c)**

$$f(x) = \sin x + \sin x\int_{-\pi/2}^{\pi/2} f(t)dt + \cos x\int_{-\pi/2}^{\pi/2} t\ f(t)\,dt$$

$f(x) = A\ \sin x + B\ \cos x$

$$\therefore \ \ A = 1 + \int_{-\pi/2}^{\pi/2} f(t)dt$$

$$= 1 + \int_{-\pi/2}^{\pi/2} (A\sin t + B\cos t)dt = 1 + 2B\int_0^{\pi/2} \cos t\,dt$$

$A = 1 + 2B$ .....(1)

$$B=\int_{-\pi/2}^{\pi/2} t.\ f(t)dt=\int_{-\pi/2}^{\pi/2} t(A\sin t+B\cos t)dt$$

$$=2A\int_{0}^{\pi/2} t\sin t\,dt$$

$B=2A$ .....(2)

Solving (1) & (2) $A=-1/3, B=-2/3$

$$\therefore\quad f(x)=-\frac{1}{3}(\sin x+2\cos x)$$

Range of $f(x)$ is $\left[\frac{-\sqrt{5}}{3},\frac{\sqrt{5}}{3}\right]$

$$\int_0^{\pi/2} f(x)dx=\frac{-1}{3}\int_0^{\pi/2}(\sin x+2\cos x)dx=-1$$

Soln.: 44. (d) 45. (b) 46. (a)

44. (d) : Let $P=\int_0^{\pi/2}\frac{\sin x\cos x}{x+1}dx$

$$=\frac{1}{2}\int_0^{\pi/2}\frac{\sin 2x}{x+1}dx=\int_0^{\pi/2}\frac{\sin\ 2x}{2x+2}dx$$

Let $2x=t$, so that $dx=\frac{1}{2}dt$

$$P=\frac{1}{2}\int_0^{\pi}\frac{\sin t}{t+2}dt=\frac{1}{2}\left\{\left[-\frac{\cos t}{t+2}\right]_0^{\pi}-\int_0^{\pi}\frac{\cos t}{(t+2)^2}dt\right\}$$

$$=\frac{1}{2}\left\{\frac{1}{2}-\frac{-1}{\pi+2}\right\}-\frac{1}{2}I=\frac{1}{4}+\frac{1}{2(\pi+2)}-\frac{1}{2}I$$

45. (b) : $\int_3^4\frac{e^{-x}}{x-5}dx$

Let $4-x=t$, to obtain $\int_3^4\frac{e^{-x}}{x-5}dx=-\int_0^1\frac{e^{t-4}}{t+1}dt$

$$=-e^{-4}\int_0^1\frac{e^t}{t+1}dt=-e^{-4}J$$

46. (a) : Rewriting the integral as

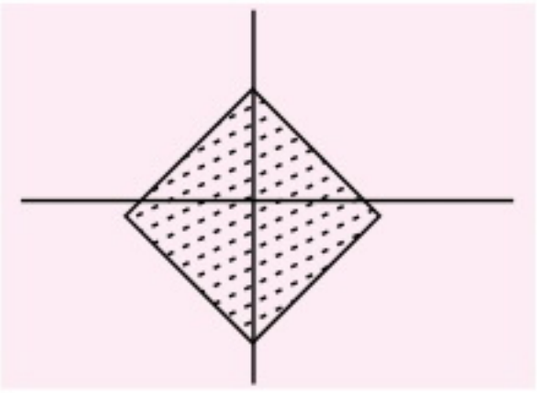
$$\int_0^1\left(\frac{x}{x+1}\right)^2e^x dx=\int_0^1\frac{(x+1)^2-2(x+1)+1}{(x+1)^2}e^x dx$$

$$=\int_0^1 e^x dx-2\int_0^1\frac{e^x}{x+1}dx+\int_0^1\frac{e^x}{(x+1)^2}dx$$

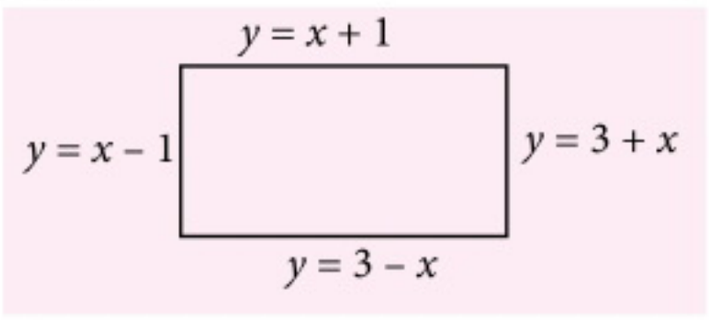
which simplifies to $\frac{e}{2}-J$

47. (A → q; B → s; C → s; D → s)

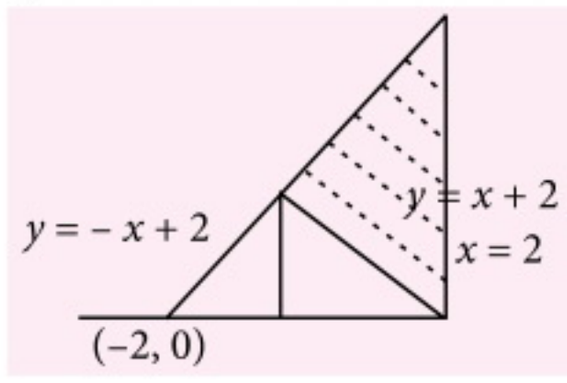
(A) Area $=4.\frac{1}{2}.1.1=2$



(B) Area = 4



(C) Area $=\frac{1}{2}.2.4=4$



(D) Area $=\int_2^4 1+\frac{8}{x^2}dx=4$

48. (A → r; B → p; C → q; D → r)

(A) $\int_0^{\frac{\pi}{2}}x^2\operatorname{cosec}^2 x dx=[x^2(-\cot x)]_0^{\pi/2}+\int_0^{\frac{\pi}{2}}2x\cot x\ dx$

$$=2\left[[x\log\sin x]_0^{\pi/2}-\int_0^{\frac{\pi}{2}}1.\log\sin x dx\right]=2\left(\frac{\pi}{2}\log 2\right)$$

$=\pi\log 2$

(B) $\tan x=t\ d(\tan x)=dt=\int\frac{2t}{1+t^2}dt$

(C) $\int_0^2 \frac{2+x}{\sqrt{4-x^2}}dx = 2\int_0^2 \frac{1}{\sqrt{4-x^2}}dx - \frac{1}{2}\int_0^2 \frac{-2x}{\sqrt{4-x^2}}dx$

$= \left[2\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2 - \left[\sqrt{4-x^2}\right]_0^2 = \pi + 2$

(D) Put $x = \sin\theta$; $2\int_0^{\frac{\pi}{2}} \frac{\theta}{\sin\theta}\cos\theta d\theta$

49. (A $\to$ s; B $\to$ r; C $\to$ r; D $\to$ q)

(A) $I_1 = \frac{4!.5!}{10!}$, $I_2 = \frac{5!.5!}{11!} \Rightarrow \frac{I_2}{I_1} = \frac{5}{11}$

(B) $I_1 = \frac{3.5.3}{10.8.6.4.2}.\pi$, $I_2 = \frac{3.3}{8.6.4.2}2\pi \Rightarrow \frac{I_1}{I_2} = \frac{1}{4}$

(C) $\int_{-a}^{a} f(x)dx = \int_0^a (f(x)+f(-x))dx$

$\therefore \quad I = \int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{\sin^2 2x}\left(\frac{1}{1-e^{-3x}} + \frac{1}{1-e^{3x}}\right)dx$

$= \frac{1}{4}\int_0^{\frac{\pi}{4}} \sec^2 x dx = \frac{1}{4}$

(D) $\tan x = t \Rightarrow I = \int_0^{\infty} \frac{dt}{(\sqrt{t}+1)^4}$ $\sqrt{t} = u-1$

$= 2\int_1^{\infty} \frac{u-1}{u^4}du = 2\int_1^{\infty}(u^{-3}-u^{-4})du = \frac{1}{3}$

50. (6) $\int_0^{1-m} (x - x^2 - mx)dx = \pm\frac{9}{2}$

$\Rightarrow \frac{(1-m)^3}{6} = \pm\frac{9}{2} \Rightarrow m = 4, -2$

51. (2) $I = -\int_0^1 ln(x).d(xe^x) = -xe^x ln(x).\int_0^1 e^x dx$

$= 0 + e - 1 = e - 1 \approx 1.7$

52. (3) $\int_{-2}^{2} e^{\cos x}\sin x + \int_2^3 2dx = 0 + 2(3-2) = 2$

53. (3) :

$$f(x) = \int_1^x (t^3 - 3t^2 + 2t)dt$$

$f'(x) = x^3 - 3x^2 + 2x = x(x-2)(x-1)$

Min $f(x) = f(2) = -\frac{1}{4}$

Max $f(x) = f(3) = 2$ Range $= \left[-\frac{1}{4}, 2\right]$

54. (2) $f''(x) = f(x)$

$\Rightarrow 2f'(x)f''(x) = 2f(x).f'(x)$

$\Rightarrow \frac{d}{dx}\left[(f'(x))^2\right] = \frac{d}{dx}\left[(f(x))^2\right]$

$\Rightarrow [f'(x)]^2 = [f(x)]^2 + C$ give $f(0) = 2$,

$f'(0) = 3$ $C = 5$

$\therefore \quad (f'(x))^2 = (f(x))^2 + 5$

$f'(x) = \sqrt{5+(f(x))^2}$

$\Rightarrow \frac{f'(x)}{\sqrt{5+(f(x))^2}} = 1$ integrating

$\Rightarrow \int \frac{d(f(x))}{(\sqrt{5})^2 + (f(x))^2} = x + C_1$

$\Rightarrow \log\left(f(x) + \sqrt{5+(f(x))^2}\right) = x + C_1$

$f(0) = 2$

$\Rightarrow C_1 = \log 5$

$\Rightarrow \log\left(f(x) + \sqrt{5+(f(x))^2}\right) = x + \log 5$

$\Rightarrow f(x) + \sqrt{5+(f(x))^2} = 5e^x$

$\therefore \quad 2f(x) = 5(e^x - e^{-x})$

$\Rightarrow f(4) = \frac{5}{2}(e^4 - e^{-4}) = \frac{5}{2}\frac{(e^8-1)}{e^4}$

$\therefore \quad k = \frac{5}{2} \Rightarrow [K] = 2$

55. (2) $\frac{d}{dx}(x\sin x + \cos x) = x\cos x$

$I = \int\left(\frac{x}{\cos x}\right)\left(\frac{x\cos x}{(x\sin x + \cos x)^2}\right)dx$ Integrate by parts

$$\left(\frac{x}{\cos x}\right)\left(\frac{-1}{x\sin x+\cos x}\right)+\int\frac{1}{\cos^2 x}dx$$

$$\frac{-x}{\cos x(x\sin x+\cos x)}+\tan x+c$$

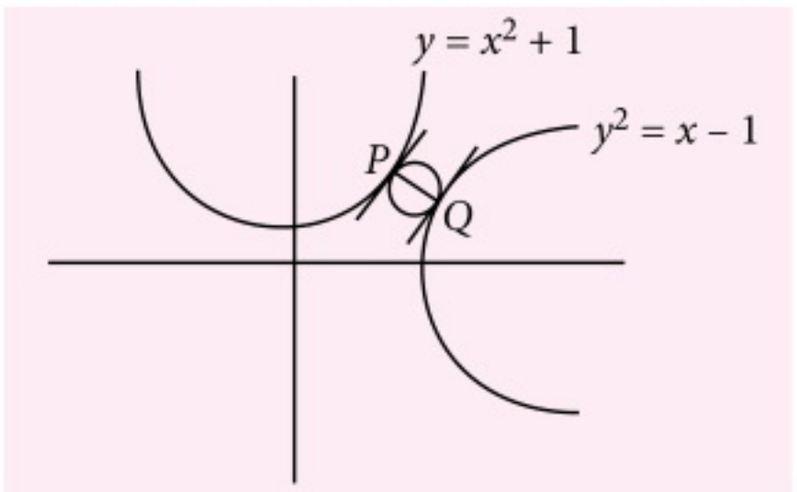
$$=\frac{\sin x-x\cos x}{x\sin x+\cos x}+c=l+p=2$$

56. (9) Required area of circle is shortest distance between two curves as diameter of circle. Shortest distance is along the common normal.

$y = x^2 + 1$, $y^2 = x - 1$ are symmetric about the line $y = x$

$\therefore$ common tangent to parabola and circles is parallel to $y = x$.

Hence: $P=\left(\frac{1}{2},\frac{5}{4}\right)$ and $Q=\left(\frac{5}{4},\frac{1}{2}\right)$



$$PQ=\frac{3\sqrt{2}}{4}\Rightarrow \text{ min. radius of circle is}$$

$$\frac{3\sqrt{2}}{8}\text{ min. area}=\frac{9\pi}{32}$$

57. (4) $I=\int\frac{\sin^2 x\cos^2 x}{(\sin x+\cos x+1)^2}dx$

$$=\frac{1}{4}\int\frac{((\sin x+\cos x)^2-1)^2}{(\sin x+\cos x+1)^2}=\frac{1}{4}\int(\sin x+\cos x-1)^2dx$$

on simplifying $(a + b + c) = 4$

58. (1) The curve $xy^2 = 4(2 - x)$ is symmetrical about x-axis and at $x \to 0, y \to \infty$

i.e. y-axis is asymptote and cuts x-axis at $x = 2$

$$\because\quad y^2=\frac{4(2-x)}{x}\geq 0$$

$$\Rightarrow\quad \frac{2-x}{x}\geq 0\Rightarrow\frac{(x-2)}{x}\leq 0$$

$\therefore\quad x\in(0,2)$

Then, $\lambda=2\int_0^2 2\sqrt{\left(\frac{2-x}{x}\right)}dx$

Put $x = 2\sin^2\theta$

$\therefore\quad dx = 4\sin\theta\cos\theta\, d\theta$

$$\Rightarrow\quad \lambda=4\int_0^{\pi/2}\sqrt{\left(\frac{2-2\sin^2\theta}{2\sin^2\theta}\right)}\times 4\sin\theta\cos\theta d\theta$$

$$=16\int_0^{\pi/2}\cos^2\theta d\theta=16.\frac{1}{2}.\frac{\pi}{2}=4\pi$$

Then, $\cos(1824\lambda) = \cos(1824, 4\pi) = 1$

59. (3) $f(x)=-\sin x\int_{\pi^2/16}^{x^2}\frac{\sin\sqrt{\theta}}{1+\cos^2\sqrt{\theta}}\cdot d\theta;$

$$f'(x)=-\sin x\left[\frac{\sin x}{1+\cos^2}\cdot 2x-0\right]+\left(\int_{\pi^2/16}^{x^2}\frac{\sin\sqrt{\theta}}{1+\cos^2\sqrt{\theta}}d\theta\right)\cos x$$

$$f'\left(\frac{\pi}{2}\right)=\frac{\pi}{1}$$

60. (0) $\int_0^1\frac{\ln(x+1)}{x^2+1}dx$ put $x=\tan\theta$

$$=\int_0^{\pi/4}\ln(1+\tan\theta)\,d\theta=\int_0^{\pi/4}\ln\left(1+\frac{1-\tan\theta}{1+\tan\theta}\right)d\theta$$

$$=\int_0^{\pi/4}\ln\frac{2}{1+\tan\theta}d\theta=\frac{\pi}{4}\ln 2-u$$

$\therefore\quad u=\frac{\pi}{8}\ln 2\Rightarrow 4u=\frac{\pi}{2}\ln 2$ ...(1)

again $v=\int_0^{\pi/2}\ln(\sin 2x)dx$ (put $2x=t$);

$$v=\frac{1}{2}\int_0^{\pi}\ln(\sin t)\,dt=\int_0^{\pi/2}\ln(\sin t)dx$$

$$v=-\frac{\pi}{2}\ln 2$$ ...(2)

(1) + (2) $\Rightarrow$ $4u + v = 0 \Rightarrow$ (C)]

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CHAPTERWISE PRACTICE PAPER

Series-5

Application of Derivatives

APPLICATION OF DERIVATIVES

RATE OF CHANGE OF QUANTITIES

- Let $y = f(x)$. Then $\frac{dy}{dx}$ denotes the rate of change of y w.r.t. x and at $x = a$ it is denoted by $\left[\frac{dy}{dx}\right]_{x=a}$
- If $x = f(t)$ and $y = g(t)$, then by chain rule, we have $\frac{dy}{dx} = \left(\frac{dy}{dt}\right) \cdot \left(\frac{dt}{dx}\right)$

MARGINAL COST AND MARGINAL REVENUE

Let C be the total cost of producing and marketing x units of a product, then marginal cost $(MC) = \frac{dC}{dx}$

The rate of change of total revenue with respect to the quantity sold is called marginal revenue, where marginal revenue $(MR) = \frac{dR}{dx}$

ERROR AND APROXIMATIONS

Let $y = f(x)$ and Δx be the small change in x and Δy be the small change in y.

Algorithm to find the approximate value of certain quantity :

1. Find Δx and x
2. $\Delta y = \frac{dy}{dx}(\Delta x)$
3. $\Delta y = f(x + \Delta x) - f(x)$ or $f(x + \Delta x) = \Delta y + f(x)$

Remark :

$\Delta y = \frac{dy}{dx}(\Delta x)$ is used in the calculation of small changes (or errors).

(i)	Absolute Error	Δx is absolute error in x
(ii)	Relative Error	$\frac{\Delta x}{x}$ is called the relative error
(iii)	Percentage Error	$\left(\frac{\Delta x}{x} \times 100\right)$ is called the percentage error

TANGENTS AND NORMALS

(i) The slope of the tangent to the curve $y = f(x)$ at (x_1, y_1) is $\left(\frac{dy}{dx}\right)_{(x_1, y_1)}$ and slope of the normal is $-\frac{1}{\left(\frac{dy}{dx}\right)_{(x_1, y_1)}}$

(ii) The equation of tangent at (x_1, y_1) is

$y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1, y_1)} (x - x_1)$ and equation of normal at (x_1, y_1) is $y - y_1 = \left(\frac{-1}{\left(\frac{dy}{dx}\right)_{(x_1, y_1)}}\right)(x - x_1)$

(iii) Slope of tangent, $m = \frac{dy}{dx} = \tan\theta$, θ is angle made by tangent with positive x-axis.

(iv) Angle between the two curves is given by $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$, where m_1 and m_2 are slopes of tangents to two curves.

- **Points to Remember**

m_1, m_2 are the slopes of two lines		
(i)	$m_1 = m_2$	Parallel lines
(ii)	$m_1 \cdot m_2 = -1$	Perpendicular lines
(iii)	$\frac{dy}{dx} = 0$ at (x_1, y_1)	Tangent is parallel to x-axis
(iv)	$\frac{dy}{dx} = \infty$ at (x_1, y_1)	Tangent is perpendicular to x-axis

INCREASING AND DECREASING FUNCTIONS (MONOTONICITY)		
1.	Without derivative test	A function $f(x)$ in the open interval (a, b) is said to be (i) Increasing function: If $x_1 < x_2 \Rightarrow f(x_1) \le f(x_2)$ for all $x_1, x_2 \in (a, b)$ (ii) Strictly increasing function: If $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$ for all $x_1, x_2 \in (a, b)$ (iii) Decreasing function: If $x_1 < x_2 \Rightarrow f(x_1) \ge f(x_2)$ for all $x_1, x_2 \in (a, b)$ (iv) Strictly decreasing function: If $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$ for all $x_1, x_2 \in (a, b)$
2.	First derivative test	Let f be continuous on $[a, b]$ and differentiable in open interval (a, b), then (i) Increasing function in $[a, b]$: If $f'(x) \ge 0$ for all $x \in (a, b)$ (ii) Strictly increasing in (a, b): If $f'(x) > 0$ for all $x \in (a, b)$ (iii) Decreasing function in $[a, b]$: If $f'(x) \le 0$ for all $x \in (a, b)$ (iv) Strictly decreasing in (a, b): If $f'(x) < 0$ for all $x \in (a, b)$ $f(x)$ is a constant funtion in $[a, b]$ if $f'(x) = 0$ for each $x \in (a, b)$
3.	Critical or Stationary point	The value of x for which $f'(x) = 0$

MAXIMA AND MINIMA	
Maximum	Let $f(x)$ be a function with domain $D \subset R$. $f(x)$ is said to attain the maximum value at a point $a \in D$, if $f(x) \le f(a)$, for all $x \in D$. a is called the point of maxima and $f(a)$ is known as the maximum value or the greatest value of $f(x)$.
Minimum	Let $f(x)$ be a function with domain $D \subset R$. $f(x)$ is said to attain the minimum value at a point $a \in D$, if $f(x) \ge f(a)$, $x \in D$. a is called the point of minima and $f(a)$ is known as the minimum value or the least value of $f(x)$.
Without derivative test	
Local maxima	$f(x)$ is said to attain local maximum value at $x = a$, if there exists a neighbourhood $(a - \delta, a + \delta)$ of a such that $f(x) < f(a)$ for all $x \in (a - \delta, a + \delta)$, $x \ne a$. $f(a)$ is called the local maximum value of $f(x)$ at $x = a$.

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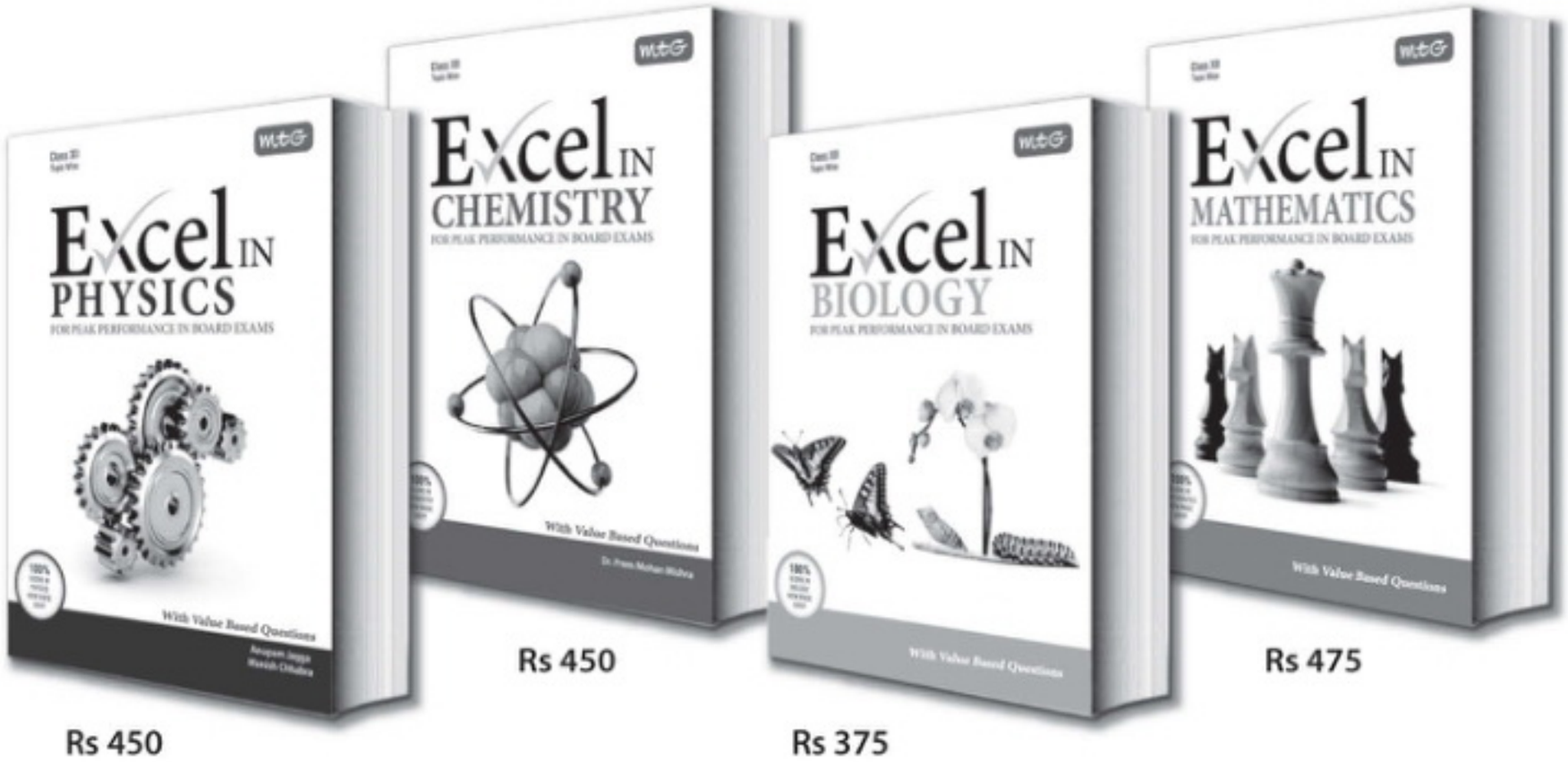

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Local Minima	$f(x)$ is said to attain local minimum value at $x = a$, if there exists a neighbourhood $(a - \delta, a + \delta)$ of a such that $f(x) > f(a)$ for all $x \in (a - \delta, a + \delta)$, $x \neq a$. $f(a)$ is called the local minimum value of $f(x)$ at $x = a$.
First derivative test	
Local Maximum	Let $f(x)$ be a function defined on an open interval I & continuous at a critical point $a \in I$, then $x = a$ is a point of local maximum of $f(x)$, if (i) $f'(a) = 0$ and, (ii) $f'(x)$ changes sign from positive to negative as x passes through a *i.e.*, $f'(x) > 0$ at every point in the left *nbd* $(a - \delta, a)$ of a and $f'(x) < 0$ at every point in the right *nbd* $(a, a + \delta)$ of a.
Local Minimum	Let $f(x)$ be a function difined on an open interval I & continuous at a critical point $a \in I$, then $x = a$ is a point of local minimum of $f(x)$, if (i) $f'(a) = 0$ and, (ii) $f'(x)$ changes sign from negative to positive as x passes through a *i.e.*, $f'(x) < 0$ at every point in the left *nbd* $(a - \delta, a)$ of a and $f'(x) > 0$ at every point in the right *nbd* $(a, a + \delta)$ of a.
Point of inflection	Let $f(x)$ be a function difined on an open interval I & continuous at a critical point $a \in I$, then If $f'(a) = 0$, but $f'(x)$ does not change sign, that is, $f'(a)$ has the same sign in the complete *nbd* of a, then a is neither a point of local maximum nor a point of local minimum.
Second derivative test	
Local maxima Local minima	Let $f(x)$ be a differentiable function defined on an open interval I and let c be an interior point of I such that (i) $x = c$ is a point of local maxima if $f'(c) = 0$ and $f''(c) < 0$ (ii) $x = c$ is a point of local minima if $f'(c) = 0$ and $f''(c) > 0$ (iii) The test fails if $f'(c) = 0$ and $f''(c) = 0$.
Absolute maxima and Absolute minimum	
Absolute maxima Absolute minima.	Let f be a differentiable function on a closed interval I and let c be any interior point of I. Then (i) $f'(c) = 0$ if f attains its absolute maximum value at c. (ii) $f'(c) = 0$ if f attains its absolute minimum value at c.

VERY SHORT ANSWER TYPE (1 MARK)

1. If the rate of change of volume of a sphere is equal to the rate of change of its radius, find the radius of the sphere.
2. An edge of a variable cube is increasing at the rate of 5 cm per second. How fast is the volume increasing when the side is 15 cm.
3. Find the slope of the tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at $t = 2$.
4. If $y = \log_e x$, find Δy when $x = 3$ and $\Delta x = 0.03$.
5. The total revenue received from the sale of x units of a product is given by $R(x) = 3x^2 + 40x + 10$. Find the marginal revenue when $x = 5$.

SHORT ANSWER TYPE (4 MARKS)

6. Find the intervals in which the function $f(x) = x^3 - 12x^2 + 36x + 17$ is (a) increasing, (b) decreasing.

7. Find the approximate value of $f(5.001)$, where $f(x) = x^3 - 7x^2 + 15$.

8. A point source of light along a straight road is at a height of a metres. A boy b metres in height is walking along the road. How fast is his shadow increasing if he is walking away from the source of light at the rate of c metres per minute.

9. If the error committed in measuring the radius of a circle be 0.01%, find the corresponding error in calculating the area.

10. Find the points of local maxima or local minima of the function

$$f(x) = (\sin^4 x + \cos^4 x) \text{ in } 0 < x < \frac{\pi}{2}.$$

LONG ANSWER TYPE (6 MARKS)

11. A wire of length 28 cm is to be cut into two pieces. One of the two pieces is to be made into square and other into a circle. What should be the length of two pieces so that the combined area of them is minimum?

12. If length of three sides of a trapezium other than base are equal to 10 cm, then find the area of the trapezium when it is maximum.

13. Show that the area of the triangle formed by the tangent and the normal at the point (a, a) on the curve $y^2(2a - x) = x^3$ and the line $x = 2a$, is $\frac{5a^2}{4}$ sq. units.

14. Find the maximum and minimum values of $(x + \sin 2x)$ on $[0, 2\pi]$.

15. An Apache helicopter of enemy is flying along the curve given by $y = x^2 + 7$. A soldier, placed at $(3, 7)$ wants to shoot down the helicopter when it is nearest to him. Find the nearest distance.

SOLUTIONS

1. We have,

$\frac{dV}{dt} = \frac{dr}{dt}$, where V is the volume and r is the radius of the sphere

$$\Rightarrow \frac{d\left(\frac{4}{3}\pi r^3\right)}{dt} = \frac{dr}{dt} \Rightarrow \frac{4}{3}\pi \times 3r^2 \frac{dr}{dt} = \frac{dr}{dt}$$

$$\Rightarrow 4\pi r^2 = 1 \Rightarrow r^2 = \frac{1}{4\pi} \quad \therefore \quad r = \frac{1}{2\sqrt{\pi}} \text{ units}$$

2. Let x be the edge of the cube and V be the volume of the cube at any time t.

Given $\frac{dx}{dt} = 5$ cm/sec, $x = 15$.

Since we know the volume of cube = $(\text{side})^3$

i.e., $V = x^3$

$$\Rightarrow \frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt}$$

$$\Rightarrow \frac{dV}{dt} = 3 \cdot (15)^2 \times 5 = 3375 \text{ cm}^3/\text{sec}$$

3. We have, $x = t^2 + 3t - 8$ & $y = 2t^2 - 2t - 5$

$$\text{Slope of the tangent} = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{t=2} = \left(\frac{4t-2}{2t+3}\right)_{t=2} = \frac{4(2)-2}{2(2)+3} = \frac{6}{7}$$

4. We have, $y = \log_e x$

$$\therefore \Delta y = \frac{dy}{dx} \cdot \Delta x = \frac{1}{x} \cdot \Delta x = \frac{0.03}{3} = 0.01$$

5. We have, $R(x) = 3x^2 + 40x + 10$

$$MR = \frac{dR}{dx} = \frac{d}{dx}(3x^2 + 40x + 10)$$

$\Rightarrow [MR]_{x=5} = (6 \times 5 + 40) = 70.$

Hence, the required marginal revenue is ₹ 70.

6. We have, $f(x) = x^3 - 12x^2 + 36x + 17$

$\Rightarrow f'(x) = 3x^2 - 24x + 36 = 3(x - 6)(x - 2)$

$\therefore f'(x) = 0$ gives $x = 2, 6$.

The points $x = 2, 6$ divides the real line into three disjoint interval; $(-\infty, 2), (2, 6), (6, \infty)$

Interval	Sign of $f(x)$	Nature of function
$(-\infty, 2)$	(−ve)(−ve) = +ve	Increasing
$(2, 6)$	(−ve)(+ve) = −ve	Decreasing
$(6, \infty)$	(+ve)(+ve) = +ve	Increasing

So, $f(x)$ is increasing $(-\infty, 2) \cup (6, \infty)$ and decreasing on $(2, 6)$.

7. Let $x = 5$ and $\Delta x = 0.001$, then

$f(5.001) = f(x + \Delta x), f'(x) = 3x^2 - 14x$

We know that,

$\Delta y = f(x + \Delta x) - f(x)$

$\Delta y = f(5.001) - (x^3 - 7x^2 + 15)$

$f(5.001) = \Delta y + (x^3 - 7x^2 + 15)$

$$= \left(\frac{dy}{dx}\right)_{x=5} \times \Delta x + ((5)^3 - 7(5)^2 + 15)$$

$= [3(5)^2 - 14(5)]\ 0.001 + (125 - 175 + 15)$

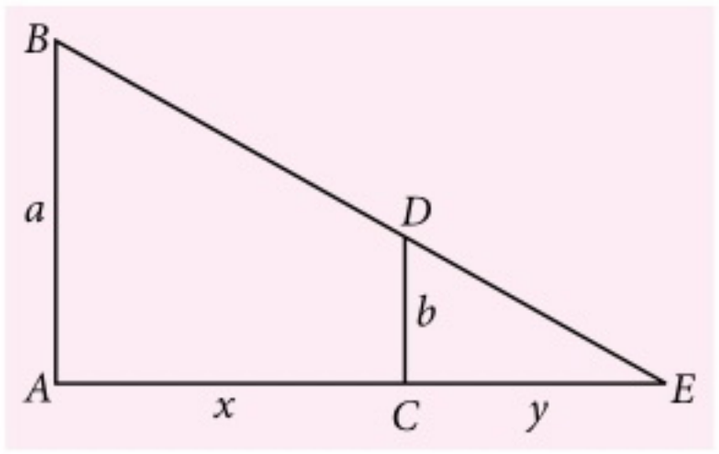
$= (75 - 70) \times 0.001 - 35$

$= 5 \times 0.001 - 35 = -34.995$

8. Let AB be the Source of light. Then, $AB = a$ metres. At any instant t, let CD be the boy and CE be his shadow. Then, $CD = b$ metres.

Let $AC = x$ metres and $CE = y$ metres

Given that $\frac{dx}{dt} = c$ metres/min.



Clearly, $\Delta BAE \sim \Delta DCE$.

$\therefore \quad \frac{AB}{CD} = \frac{AE}{CE}$

$\Rightarrow \quad \frac{a}{b} = \frac{x+y}{y}$

$\Rightarrow \quad (a - b)y = bx$

$\Rightarrow \quad (a-b)\frac{dy}{dt} = b.\frac{dx}{dt} = bc \qquad \left[\because \frac{dx}{dt} = c\right]$

$\Rightarrow \quad \frac{dy}{dt} = \frac{bc}{(a-b)}$ m/min.

Hence, the shadow is increasing at the rate of $\frac{bc}{(a-b)}$ m/min.

9. $A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r \Rightarrow \frac{dA}{dr} \cdot \delta r = 2\pi r \cdot \delta r$

$\Rightarrow \delta A = 2\pi r^2 \cdot \frac{\delta r}{r} \qquad [\because \frac{dA}{dr} \cdot \delta r = \delta A]$

$\Rightarrow \frac{\delta A}{A} = 2 \cdot \frac{\delta r}{r} \qquad [\because A = \pi r^2]$

$\Rightarrow \left(\frac{\delta A}{A} \times 100\right) = 2 \cdot \left(\frac{\delta r}{r} \times 100\right)$

$\Rightarrow$ percentage error in area $= (2 \times 0.01)$

$= 0.02\%$

10. $f(x) = \sin^4 x + \cos^4 x$

$\Rightarrow \ f'(x) = 4\sin^3 x \cos x - 4\cos^3 x \sin x$

$= -4 \sin x \cos x (\cos^2 x - \sin^2 x)$

$= -2 \sin 2x \cos 2x = -\sin 4x.$

And $f''(x) = -4 \cos 4x.$

Now, $f'(x) = 0 \Rightarrow -\sin 4x = 0 \Rightarrow \ 4x = \pi,$

i.e., $x = \frac{\pi}{4}$.

$\therefore\ x = (\pi/4)$ is a point of local maximum or local minimum.

Now, $f''\left(\frac{\pi}{4}\right) = -4 \cos \pi = 4 > 0.$

$\therefore\ x = (\pi/4)$ is a point of local mininum.

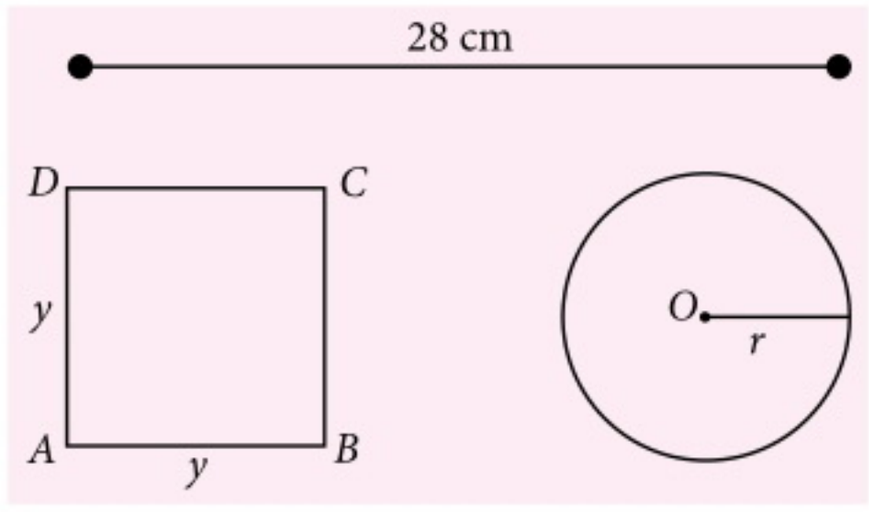
Local minimum value $= f\left(\frac{\pi}{4}\right) = \frac{1}{2}$.

11. Let the length of one piece be x, then the length of other piece will be $28 - x$.

Let from the first piece we make a circle of radius r units and from the second piece we make a square of side y units.

$\Rightarrow \quad 2\pi r = x \Rightarrow r = \frac{x}{2\pi}$ and

$4y = 28 - x \Rightarrow y = \frac{(28-x)}{4} \qquad ...(i)$



Let A be the combined area of the circle and square.

Then, $A = \pi r^2 + y^2$

$\Rightarrow \quad A = \pi\left(\frac{x}{2\pi}\right)^2 + \left(\frac{28-x}{4}\right)^2 \qquad ...(ii)$

Differentiating (ii) with respect to 'x', we get

$$A' = \frac{\pi}{4\pi^2}(2x) + \frac{1}{16}(2)(28-x)(-1)$$

$$\Rightarrow \quad A' = \frac{x}{2\pi} - \frac{1}{8}(28-x)$$

$$\text{and } A'' = \frac{1}{2\pi} - \frac{1}{8}(0-1) = \frac{1}{2\pi} + \frac{1}{8}$$

For maximum and minimum $A' = 0$

$$\Rightarrow \quad \frac{x}{2\pi} - \frac{28}{8} + \frac{x}{8} = 0$$

$$\Rightarrow \quad x\left(\frac{1}{2\pi} + \frac{1}{8}\right) = \frac{28}{8} \quad \Rightarrow \quad x\left(\frac{4+\pi}{8\pi}\right) = \frac{28}{8}$$

$$\Rightarrow \quad x = \frac{28\pi}{4+\pi}$$

Since A'' is $+ve$ for $x = \frac{28\pi}{4+\pi}$

$$\therefore \quad A \text{ is minimum for } x = \frac{28\pi}{4+\pi}$$

Thus, the required length of two pieces are

$$x = \frac{28\pi}{4+\pi} \text{ cm}$$

$$\text{and } 28 - x = 28 - \frac{28\pi}{4+\pi} = \frac{112}{4+\pi} \text{ cm.}$$

12. The required trapezium is as given in figure.
Draw perpendiculars DP and CQ on AB.
Let $AP = x$ cm.
Since $\Delta PAD \cong \Delta BQC$ by RHS property, therefore $AP = QB$
$\Rightarrow$ $QB = x$ cm.
Also, by Pythagoras theorem

$$DP = QC = \sqrt{100 - x^2}.$$

Let A be the area of the trapezium

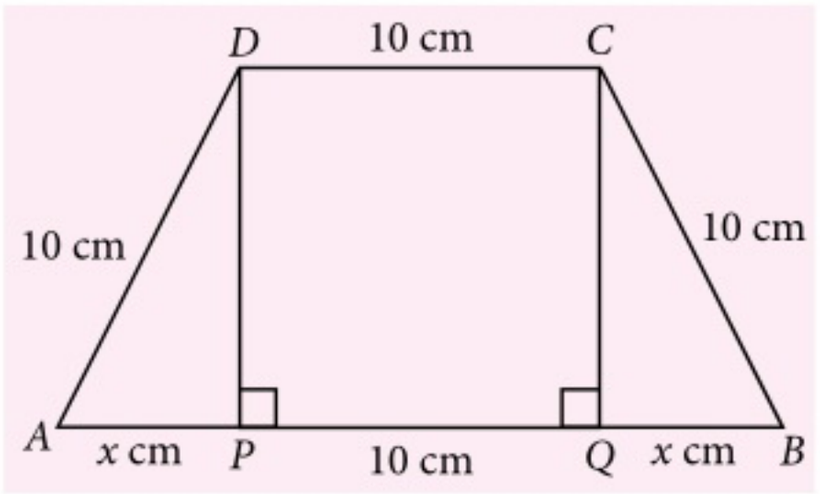
$$\text{then, } A = \frac{1}{2}(\text{sum of parallel side}) \times (\text{height})$$

$$= \frac{1}{2}(2x + 10 + 10)(\sqrt{100 - x^2})$$

$$= (x+10)\sqrt{100 - x^2}$$

$$\text{or, } A'(x) = (x+10)\frac{(-2x)}{2\sqrt{100-x^2}} + (\sqrt{100-x^2})$$

$$= \frac{-2x^2 - 10x + 100}{\sqrt{100-x^2}}$$



Now, $A'(x) = 0$ gives $2x^2 + 10x - 100 = 0$,
i.e., $x = 5$ and $x = -10$.
Since x represents length, it cannot be negative.
So, $x = 5$.
Now,

$$A''(x) = \frac{\sqrt{100-x^2}(-4x-10) - (-2x^2-10x+100)\frac{(-2x)}{2\sqrt{100-x^2}}}{100-x^2}$$

$$= \frac{2x^3 - 300x - 1000}{(100-x^2)^{\frac{3}{2}}} \text{ (on simplification)}$$

or

$$A''(5) = \frac{2(5)^3 - 300(5) - 1000}{(100-(5)^2)^{\frac{3}{2}}} = \frac{-2250}{75\sqrt{75}} = \frac{-30}{\sqrt{75}} < 0$$

Thus, area of trapezium is maximum at $x = 5$ and the maximum area is

$$(5+10)\sqrt{100-(5)^2} = 15\sqrt{75} = 75\sqrt{3} \text{ cm}^2$$

13. The given curve is $y^2(2a - x) = x^3$ ...(i)
and given point is (a, a).
Now, differentiating (i) w.r.t. x, we get

$$(2a-x)\cdot 2y\cdot\frac{dy}{dx} - y^2\cdot 1 = 3x^2 \Rightarrow \frac{dy}{dx} = \frac{3x^2+y^2}{(2a-x)2y}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(a,a)} = \frac{3a^2+a^2}{(2a-a)\cdot 2a} = \frac{4a^2}{2a^2} = 2$$

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Let the slope of tangent be m_1 *i.e.*, $m_1 = 2$, then slope of normal will be $-\frac{1}{2} = m_2$

$\therefore$ Equation of tangent at point (a, a) having slope 2 is

$$y - a = 2(x - a) \Rightarrow 2x - y = a$$

and equation of normal at point (a, a) having slope $-\frac{1}{2}$ is $y - a = -\frac{1}{2}(x - a) \Rightarrow x + 2y = 3a$

To find the area of triangle formed by lines $x + 2y = 3a$; $2x - y = a$ and given line $x = 2a$, first we have to find the coordinates of vertices obtained by the given lines.

Now, solving the equation of lines simultaneously, we get the coordinates of vertices as (a, a), $(2a, 3a)$ and $\left(2a, \frac{a}{2}\right)$.

$$\text{Area} = \text{absolute value of } \frac{1}{2}\begin{vmatrix} a & a & 1 \\ 2a & 3a & 1 \\ 2a & \frac{a}{2} & 1 \end{vmatrix}$$

Hence area

$$= \frac{1}{2}\left|\left[a\left(3a - \frac{a}{2}\right) + 2a\left(\frac{a}{2} - a\right) + 2a(a - 3a)\right]\right|$$

$$= \frac{5a^2}{4} \text{ sq. units.}$$

14. Let $f(x) = (x + \sin 2x)$.

Then, $f'(x) = (1 + 2\cos 2x)$ and

$f''(x) = -4 \sin 2x$.

Now, $f'(x) = 0 \Rightarrow \cos 2x = -\frac{1}{2}$,

Where $x \in [0, 2\pi)$,

$$\Rightarrow 2x = \frac{2\pi}{3} \text{ or } 2x = \frac{4\pi}{3}$$

$$\Rightarrow x = (\pi/3) \text{ or } x = (2\pi/3).$$

$$f''\left(\frac{\pi}{3}\right) = -4\sin\left(\frac{2\pi}{3}\right) = -4\sin\frac{\pi}{3} = -2\sqrt{3} < 0.$$

$\therefore$ $x = (\pi/3)$ is a point of local maxima.

Again, we consider the point $x = (2\pi/3)$.

$$f''\left(\frac{2\pi}{3}\right) = -4\sin\left(2 \times \frac{2\pi}{3}\right)$$

$$= -4\sin\frac{4\pi}{3} = 4\sin\frac{\pi}{3} = 2\sqrt{3} > 0.$$

So, $x = \frac{2\pi}{3}$ is a point of local minima.

Now, $f\left(\frac{\pi}{3}\right) = \frac{2\pi + 3\sqrt{3}}{6}$ and

$$f\left(\frac{2\pi}{3}\right) = \frac{2\pi}{3} + \sin\frac{4\pi}{3} = \frac{2\pi}{3} - \sin\frac{\pi}{3} = \frac{4\pi - 3\sqrt{3}}{6};$$

$f(0) = 0$ and $f(2\pi) = 2\pi + \sin 4\pi = 2\pi$.

$\therefore$ Minimum value of $f(x)$ is 0 at $x = 0$ and

$\therefore$ maximum value of $f(x)$ is 2π at $x = 2\pi$.

15. For each value of x, the helicopter's position is at point $(x, x^2 + 7)$.

Therefore, the distance between the helicopter and the soldier placed at (3, 7) is

$$\sqrt{(x-3)^2 + (x^2 + 7 - 7)^2} \Rightarrow \sqrt{(x-3)^2 + x^4}.$$

Let, $f(x) = (x - 3)^2 + x^4$

$\Rightarrow f'(x) = 2(x - 3) + 4x^3$

$= 2(x - 1)(2x^2 + 2x + 3)$

Thus, $f'(x) = 0$ gives $x = 1$ or $2x^2 + 2x + 3 = 0$ for which there are no real roots. Also, there are no end points of the interval to be added to the set for which $f'(x)$ is zero. *i.e.*, there is only one point, namely, $x = 1$. The value of f at this point is given by $f(1) = (1 - 3)^2 + (1)^4 = 5$. Thus, the distance between the soldier and the helicopter is $\sqrt{f(1)} = \sqrt{5}$.

Now, $f''(x) = 2 + 12x^2 > 0 \quad \forall\ x \in R$

$\therefore$ $f(x)$ has minimum value at $x = 1$.

Hence, $\sqrt{5}$ is the minimum distance between the soldier and the helicopter.

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Binomial Theorem : Binomial theorem for a positive integral index, properties of binomial coefficients, General term, Middle term.
Co-ordinate Geometry-2D : Equation of a parabola, ellipse and hyperbola in standard form, their foci, directrices and eccentricity, parametric equations, equations of tangent & normal.

PAPER-I

SECTION-I

Single Option Correct

This section contains 10 multiple choice questions. Each question has 4 choices, out of which ONLY ONE is correct.

1. The equation of directrix of the parabola $y^2 + 4y + 4x + 2 = 0$ is
(a) $x = -1$ (b) $x = 1$
(c) $x = -3/2$ (d) $x = 3/2$

2. The axis of the parabola $y^2 + 4x + 2y - 11 = 0$ is given by
(a) $y + 1 = 0$ (b) $x + 1 = 0$
(c) $y = 0$ (d) $x = 0$

3. If B, B' are the ends of minor axis and S, S' are foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$, then the area of the rhombus $SBS'B'$ formed will be
(a) 12 sq. units (b) 24 sq. units
(c) 36 sq. units (d) 48 sq. units

4. If the line joining foci of an ellipse subtends an angle 90° at an extremity of minor axis, then the eccentricity is
(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{3}$ (d) $\frac{1}{\sqrt{3}}$

5. If t is a parameter, then $x=a\left(t+\frac{1}{t}\right)$, $y=b\left(t-\frac{1}{t}\right)$ represents
(a) an ellipse (b) a circle
(c) a pair of straight lines
(d) a hyperbola

6. For different values of α, the locus of the point of intersection of the two straight lines $\sqrt{3}x - y - 4\sqrt{3}\alpha = 0$ and $\sqrt{3}\alpha x + \alpha y - 4\sqrt{3} = 0$ is
(a) a hyperbola with eccentricity 2
(b) an ellipse with eccentricity $\sqrt{\frac{2}{3}}$
(c) a hyperbola with eccentricity $\sqrt{\frac{19}{16}}$
(d) an ellipse with eccentricity $\frac{3}{4}$

7. If $(1 - x + x^2)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$, then the value of $a_0 + a_2 + a_4 + ... + a_{2n}$ is
(a) $3^n + \frac{1}{2}$ (b) $3^n - \frac{1}{2}$
(c) $\frac{3^n - 1}{2}$ (d) $\frac{3^n + 1}{2}$

8. If co-efficients of r^{th}, $(r+1)^{th}$ and $(r+2)^{th}$ terms in the expansion of $(1 + y)^m$ are in A.P., then
(a) $m^2 - m(4r - 1) + 4r^2 - 2 = 0$
(b) $m^2 - m(4r + 1) + 4r^2 + 2 = 0$
(c) $m^2 - m(4r + 1) + 4r^2 - 2 = 0$
(d) $m^2 - m(4r - 1) + 4r^2 + 2 = 0$

9. The term independent of x in the expansion of $(1+x)^m\left(1+\frac{1}{x}\right)^n$ is

By : Sankar Ghosh, HOD(Math), Takshyashila. Mob : 09831244397

(a) $\frac{(mn)!}{(m+n)!}$ (b) $\frac{(m+n)!}{(mn)!}$

(c) $\frac{m!n!}{(m+n)!}$ (d) $\frac{(m+n)!}{m!n!}$

10. The co-efficient of the middle term in the expansion of $(1 + \alpha x)^4$ and of $(1 - \alpha x)^6$ is the same if α equals

(a) $\frac{-3}{10}$ (b) $\frac{10}{3}$ (c) $\frac{-5}{3}$ (d) $\frac{3}{5}$

SECTION-II

Multiple Correct Options

This section contains 5 multiple choice questions. Each question has 4 choices, out of which ONE or MORE is/are correct.

11. If r^{th} term, $(r + 1)^{th}$ term and $(r + 2)^{th}$ term are in A.P., of $(1 + x)^{14}$, then the value of r is

(a) 5 (b) 12 (c) 10 (d) 9

12. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, $n \in N$ then $C_0 - C_1 + C_2 - .... + (-1)^{m-1} \cdot C_{m-1} =$

(a) $\frac{(n-1)(n-2)....(n-m+1)}{(m-1)!}(-1)^{m-1}$

(b) ${}^{n-1}C_{m-1}(-1)^{m-1}$

(c) $\frac{(n-1)(n-2)....(n-m)}{(m-1)!}$

(d) ${}^{n-1}C_{n-m}(-1)^{m-1}$

13. If the equation of the directrix of $x^2 + 4y - 6x + k = 0$ be $y + 1 = 0$, then

(a) $k = 17$ (b) $k = -17$

(c) focus is $(3, -3)$ (d) vertex is $(3, -3)$

14. If the tangent drawn from the point $(0, 2)$ to the parabola $y^2 = 4ax$ are inclined at an angle $\frac{3\pi}{4}$, then the value of a is

(a) 2 (b) –2

(c) 1 (d) none of these

15. A point on the ellipse $x^2 + 3y^2 = 37$, where the normal is parallel to the line $6x - 5y = 2$ is

(a) $(5, -2)$ (b) $(5, 2)$

(c) $(-5, 2)$ (d) $(-5, -2)$

SECTION-III

Assertion & Reason Type

In each of the following questions two statements are given Statement-1 as Assertion (A) and Statement-2 Reason (R). Examine the statements carefully and answer the questions according to the instruction given below:

(a) If both A and R is correct and R is a proper reason of A.

(b) If both A and R is correct and R is not the proper reason of A.

(c) If A is correct and R is wrong.

(d) If A is wrong and R is correct.

16. **Statement-1 :** The rectangular hyperbola $xy = 4^2$ does not intersect the parabola $y^2 = 4x$ at any real point.

Statement-2 : If the hyperbola $\frac{x^2}{b^2} - \frac{y^2}{a^2} = 1$ passes through the focus of the ellipse $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, then the focus of the hyperbola is $\sqrt{3}$.

17. The curve C is the locus of a point whose sum of differences from points $S(\sqrt{3}, 0)$ and $S'(\sqrt{3}, 0)$ is 4.

Statement-1 : The curve C cuts off intercept $2\sqrt{3}$ from the line $2y - 1 = 0$.

Statement-2 : The equation of the curve C is $\frac{x^2}{4} + 2y^2 = \frac{5}{4}$.

18. **Statement-1 :** If PQ is a chord of the parabola $x^2 + 2x + 12y - 11 = 0$ through $(-1, -2)$ and a circle is described on PQ as diameter, then the circle touches the line $y = 4$.

Statement-2 : If a cirlce is described on a focal chord of a parabola as diameter, then it touches the directrix.

19. **Statement-1 :** $3^{2n+2} - 8n - 9$ is divisible by 64. $(\forall\, n \in N)$

Statement-2 : $(1 + x)^2 - nx - 1$ is divisible by x^2 $(\forall\, n \in N)$

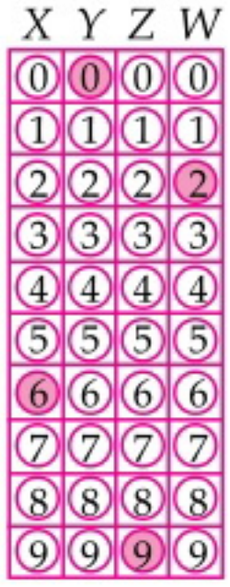
20. **Statement-1 :** The term independent of x in the expansion $\left(x + \frac{1}{x} + 2\right)^{25}$ is ${}^{50}C_{25}$.

Statement-2 : Middle term in the binomial expansion is independent of x.

SECTION-IV

Integer Answer type

This section contains 5 questions. Theanswer to each of the questions is a single-digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like as given.



21. If $\frac{T_2}{T_3}$ in $(a+b)^n$ and $\frac{T_3}{T_4}$ in $(a+b)^{n+3}$ are equal, then the value of n is

22. For what value of λ, the equation $\lambda x^2 + 4xy + y^2 + \lambda x + 3y + 2 = 0$ represents the equation of a parabola?

23. The product of the perpendiculars drawn from the foci on the tangent at any point on the ellipse $\frac{x^2}{9}+\frac{y^2}{25}=1$ is

24. Suppose the major and minor axis of an ellipse are respectively along x-axis and y-axis. The ellipse intersects the line $\frac{x}{7}+\frac{y}{2}=1$ on x-axis and $\frac{x}{3}-\frac{y}{5}=1$ on y-axis. If the eccentricity of the ellipse is $\frac{2\sqrt{6}}{\lambda}$, then the value of λ is

25. If a rectangular hyperbola whose centre is C is cut by any circle of radius r in four points P, Q, R and S, then $CP^2 + CQ^2 + CR^2 + CS^2 = \lambda r^2$, then the value of λ is

PAPER-II

SECTION-I

Single Option Correct

This section contains 10 multiple choice questions numbered 1 to 4. Each question has 4 choices (a), (b), (c)and (d), out of which ONLY ONE is correct.

1. The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the points P, Q and the parabola at the points R, S. Then the area of the quadrilateral $PQRS$ is

(a) 3 (b) 6 (c) 9 (d) 15

2. The point on the parabola $y^2 = 64x$ which is nearest to the line $4x + 3y + 35 = 0$ has coordinates

(a) $(9, -24)$ (b) $(1, 81)$
(c) $(4, -16)$ (d) $(-9, -24)$

3. The equation of the common tangent with positive slope to the parabola $y^2 = 8\sqrt{3}x$ and hyperbola $4x^2 - y^2 = 4$ is

(a) $y=\sqrt{6}x+\sqrt{2}$ (b) $y=\sqrt{6}x-\sqrt{2}$
(c) $y=\sqrt{3}x+\sqrt{2}$ (d) $y=\sqrt{3}x-\sqrt{2}$

4. Let the equation of an ellipse be $\frac{x^2}{144}+\frac{y^2}{25}=1$. Then the radius of the circle with centre $(0,\sqrt{2})$ and passing through the foci of the ellipse is

(a) 9 (b) 7 (c) 11 (d) 5

5. If $y = 4x + 3$ is parallel to a tangent to the parabola $y^2 = 12x$, then its distance from the normal to the given line is

(a) $\frac{213}{\sqrt{17}}$ (b) $\frac{219}{\sqrt{17}}$ (c) $\frac{211}{\sqrt{17}}$ (d) $\frac{210}{\sqrt{17}}$

6. The equation of the circle passing through the points of intersection of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ is

(a) $x^2 + y^2 = a^2$ (b) $x^2+y^2=\frac{a^2b^2}{a^2+b^2}$
(c) $x^2 + y^2 = b^2$ (d) $x^2+y^2=\frac{2a^2b^2}{a^2+b^2}$

7. The co-efficient of x^{11} in $(1 + 3x + 2x^2)^6$ is

(a) 144 (b) 288 (c) 216 (d) 576

8. If $n\ (>1) \in N$, then the value of $a - {}^nC_1(a-1) + {}^nC_2(a-2) + \ldots + (-1)^n.(a-n)$ is

(a) 0 (b) 1 (c) a^n (d) $(-a)^n$

9. The sum of the rational terms in $(\sqrt{2}+3^{1/5})^{10}$ is
(a) 41 (b) 32
(c) 18 (d) 82

10. The co-efficient of x^{24} in $(1+x^2)^{12}(1+x)^{12}(1+x^{24})$ is
(a) ${}^{12}C_6+1$ (b) ${}^{12}C_6$
(c) ${}^{12}C_6+3$ (d) ${}^{12}C_6+2$

SECTION-II

Multiple Correct Type

This section contains 5 multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONE OR MORE is/are correct.

11. If $Ay^2+By+Cx+D=0$ is the equation of a parabola, then its
(a) length of latus-rectum is $\left|\frac{C}{A}\right|$
(b) axis is vertical
(c) ordinate of the focus is $-\frac{B}{2A}$
(d) abscissa of focus $\frac{D}{A}+\frac{B^2}{4AC}$

12. Let A and A' are the vertices of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, whose foci S and S' and eccentricity e. Let P is a point on the ellipse such that $\frac{\Delta AA'P}{\Delta SS'P}=K$, then
(a) $0<K<1$ (b) $K>1$
(c) K, is only dependent on e
(d) K, is only dependent on b and e

13. Which one of the followings related to the hyperbola $\frac{x^2}{\cos^2\alpha}-\frac{y^2}{\sin^2\alpha}=1\left(0<\alpha<\frac{\pi}{2}\right)$ is dependable on α ?
(a) eccentricity (b) abscissa of focus
(c) directrix (d) vertex

14. If the value of the middle term of $\left(\frac{x}{2}+2\right)^8$ is 1120, then the real value of x is
(a) –2 (b) 3
(c) –3 (d) 2

15. The co-efficient of x^r in $(1+bx^2+cx^3)^n$ is a_r. Which of the followings are true?
(a) co-efficient of x^2 is nb
(b) co-efficient of x^4 is $\frac{n(n-1)}{2}b^2$
(c) $\frac{2a^4}{a_2^2}=\frac{n-1}{n}$
(d) $a_2+a_4=nb\left[1+\frac{(n+1)b}{2}\right]$.

SECTION-III

Linked Comprehension Type

This section contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE OR MORE is/are correct.

$P_{16\text{-}18}$: If a, b are prime numbers, $n\in N$, then the rational terms in the expansion of $(a^{1/p}+b^{1/q})^n$ are the terms in which indices of a and b are integral numbers. On the basis of the above information answer the following questions.

16. In the expansion of $(\sqrt{2}+\sqrt[5]{3})^{120}$ the number of irrational terms is
(a) 12 (b) 13 (c) 108 (d) 54

17. In the expansion $(2^{1/5}+7^{1/7})^{105}$ the sum of rational terms is the sum of the terms
(a) $T_1+T_{35}+T_{70}+T_{105}$
(b) $T_1+T_{36}+T_{71}+T_{106}$
(c) $T_1+T_{34}+T_{69}+T_{104}$
(d) none of these

18. In the expansion of $(7^{1/3}+11^{1/9})^{5832}$, the number of terms free from radicals is
(a) 649 (b) 648 (c) 72 (d) 647

$P_{19\text{-}21}$: If a source of light is placed at the fixed point of a parabola and if the parabola is a reflecting surface, then the ray bounce back in a line parallel to the axis of the parabola.

19. A ray of light is coming along the line $y=4$ from the positive direction of x-axis and strikes a concave mirror whose intersection with the plane XOY is a parabola $y^2=16x$, then equation of reflected ray is
(a) $3x+2y=16$ (b) $4x+3y=16$
(c) $5x+4y=9$ (d) none of these

20. A ray of light moving parallel to x-axis gets reflected from the parabolic mirror whose equation is given by $y^2 - 10y - 8x + 41 = 0$, after reflection, the ray must passes through the point

(a) $(4, 5)$ (b) $(-4, -5)$

(c) $(-4, 5)$ (d) $(4, -5)$

21. A ray of light is coming along the line $x = 2$ from positive direction of y-axis and strikes a concave mirror whose intersection with XOY plane is a parabola $x^2 = 4y$, then equation of reflected ray after the second reflection is

(a) $y + 2 = 0$ (b) $y + 1 = 0$

(c) $x + 2 = 0$ (d) none of these

SECTION-IV

Matrix-Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r and s. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example. If the correct matches are A-p and s ; B-q and r; C-p and q; and D-s, then the correct darkening of bubbles will look like the above.

	p	q	r	s
A	p	q	r	s
B	p	q	r	s
C	p	q	r	s
D	p	q	r	s

22. Match the following:

Column I		Column II	
(A)	If two consecutive terms of the expansion $(1 + x)^n$ are equal, then n is	(p)	9
(B)	If $15^n + 23^n$ is always divisible by 19, then n is	(q)	10
(C)	If ${}^{10}C_0 \cdot {}^{20}C_{10} - {}^{10}C_1 \cdot {}^{18}C_{10} + {}^{10}C_2 \cdot {}^{16}C_{10}$,... is divisible by 2^n, then n is	(r)	64
(D)	If the co-efficient of T_r, T_{r+1}, T_{r+2} of $(1 + x)^{14}$ are in A.P, then	(s)	11

23. Match the following:

Column I		Column II	
(A)	The locus of the point of intersection of the straight lines $\sqrt{3}x - y - 4\sqrt{3}\alpha = 0$ and $\sqrt{3}\alpha x + \alpha y - 4\sqrt{3} = 0$ is a hyperbola, where α is parameter. The eccentricity of hyperbola is	(p)	$\sqrt{2}$
(B)	The distance between foci of the hyperbola $7x^2 - 9y^2 = 63$ is	(q)	10
(C)	If the transverse and conjugate axis of a hyperbola are same in length, then its eccentricity is	(r)	2
(D)	The difference of the distances of a point on $\frac{x^2}{25} - \frac{y^2}{9} = 1$ from the foci is	(s)	8

SOLUTIONS

PAPER-I

1. **(d) :** The given equation of the parabola is

$y^2 + 4y + 4x + 2 = 0 \Rightarrow y^2 + 4y + 4 = -4x + 2$

$\Rightarrow (y+2)^2 = -4\left(x - \frac{1}{2}\right)$

Let $Y = y + 2$ and $X = x - \frac{1}{2}$

$\therefore Y^2 = -4X$

$\therefore$ Equation of directrix, $X = a$

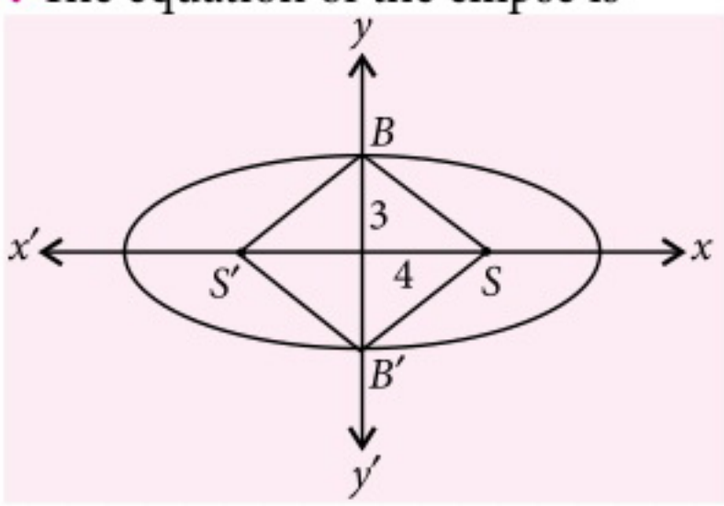
$\Rightarrow x - \frac{1}{2} = 1 \Rightarrow x = \frac{3}{2}$

2. **(a) :** The given equation of the parabola is

$y^2 + 4x + 2y - 11 = 0 \Rightarrow y^2 + 2y + 1 = -4x + 12$

$(y + 1)^2 = -4(x - 3)$ $\therefore$ axis $y + 1 = 0$

3. **(b) :** The equation of the ellipse is



$$\frac{x^2}{25}+\frac{y^2}{9}=1\ ;\quad a=5,\ b=3$$

$$e=\sqrt{1-\frac{9}{25}}=\frac{4}{5}$$

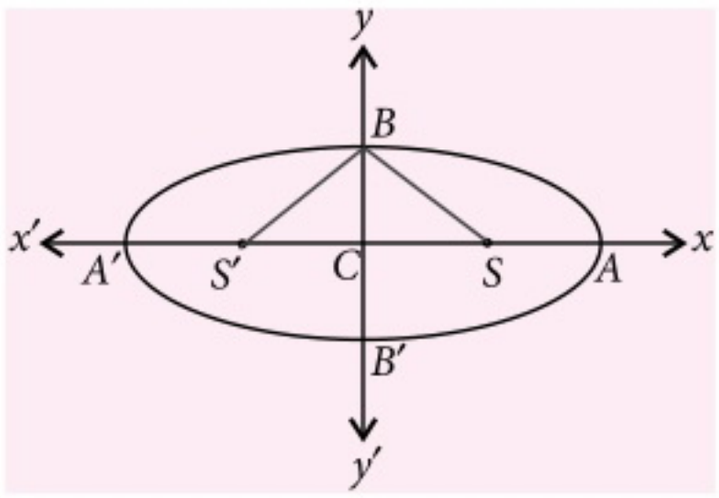
$S\equiv\left(5\times\frac{4}{5},\ 0\right)$ and $S'\equiv\left(-5\times\frac{4}{5},\ 0\right)$

i.e., $S\equiv(4,\ 0)$ and $S'\equiv(-4,\ 0)$

$B\equiv(0,\ 3)$ and $B'\equiv(0,\ -3)$

$\therefore$ Area of the rhombus $=\frac{1}{2}\times8\times6=24$ sq. units

4. (b) :



Let the equation of the ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ (a>b)$$

$\therefore\quad S\equiv(ae,\ 0),\ S'=(-ae,\ 0)$ and $B\equiv(0,\ b)$

Slope of $SB=\dfrac{0-b}{ae-0}=-\dfrac{b}{ae}$

Slope of $SB'=\dfrac{-b}{-ae-0}=\dfrac{b}{ae}$

By the problem, we have $\dfrac{-b^2}{a^2e^2}=-1$

$$\Rightarrow\ e^2=\frac{b^2}{a^2}=1-e^2\ \Rightarrow\ 2e^2=1\Rightarrow\ e^2=\frac{1}{2}$$

$$\therefore\ e=\frac{1}{\sqrt{2}}$$

5. (d) : $x=a\left(t+\dfrac{1}{t}\right)$ and $y=b\left(t-\dfrac{1}{t}\right)$

$$\therefore\ \frac{x^2}{a^2}-\frac{y^2}{b^2}=\left(t+\frac{1}{t}\right)^2-\left(t-\frac{1}{t}\right)^2=4t\cdot\frac{1}{t}=4$$

Thus, the given parametric equation represents a hyperbola.

6. (d) : The given lines are

$2x\sec\theta+y\tan\theta-2=0$

and $2x\tan\theta+y\sec\theta-1=0$

$$\frac{x}{-\tan\theta+2\sec\theta}=\frac{y}{-4\tan\theta+2\sec\theta}$$

$$=\frac{1}{2\sec^2\theta-2\tan^2\theta}$$

$$\Rightarrow\quad x=\frac{2\sec\theta-\tan\theta}{2},\ y=\frac{2(\sec\theta-2\tan\theta)}{2}$$

$\Rightarrow\ (2x)^2-y^2=(2\sec\theta-\tan\theta)^2-(\sec\theta-2\tan\theta)^2$

$=4\sec^2\theta+\tan^2\theta-4\sec\theta\tan\theta-\sec^2\theta-4\tan^2\theta$

$+4\sec\theta\tan\theta$

$=3(\sec^2\theta-\tan^2\theta)$

$(2x)^2-y^2=3\quad\therefore\quad\dfrac{4x^2}{3}-\dfrac{y^2}{3}=1$

which is a hyperbola, its eccentricity

$$=\sqrt{1+\frac{9\times16}{9}}=\sqrt{1+16}=\sqrt{17}$$

7. (d) : Given that

$(1-x+x^2)^n=a_0+a_1x+a_2x^2+\ldots.+a_{2n}x^n$

Putting $x=1$, we get

$a_0+a_1+a_2+\ldots+a_{2n}=1$ ... (i)

Putting $x=-1$, we get

$a_0-a_1+a_2-\ldots+a_{2n}=3^n$ ... (ii)

Adding (i) and (ii), we get

$2(a_0+a_2+a_4+\ldots+a_{2n})=3^n+1$

$$\therefore\quad a_0+a_2+a_4+\ldots.+a_{2n}=\frac{3^n+1}{2}$$

8. (c): $t_r=t_{(r-1)+1}={}^mC_{r-1}\,y^{r-1}$

$t_{r+1}={}^mC_r\,y^r$

$t_{r+2}={}^mC_{r+1}\,y^{r+1}$

According to the problem, we have

$2\,{}^mC_r={}^mC_{r-1}+{}^mC_{r+1}$

$$\Rightarrow\quad\frac{m!}{(r-1)!(m-r+1)!}+\frac{m!}{(r+1)!(m-r-1)!}=2\cdot\frac{m!}{r!(m-r)!}$$

$$\Rightarrow\quad\frac{r(r+1)}{(r+1)!(m-r+1)!}+\frac{(m-r+1)(m-r)}{(r+1)!(m-r+1)!}$$

$$=2\frac{(r+1)(m-r+1)}{(r+1)!(m-r+1)!}$$

$\Rightarrow\ r(r+1)+(m-r+1)(m-r)=2(r+1)(m-r+1)$

$\Rightarrow\ r(r+1)+(m-r)^2+m-r-2(r+1)(m-r+1)=0$

$\Rightarrow\ r(r+1)+m^2+r^2-2mr+m-r+2(r^2-1)$

$-2m(r+1)=0$

$\Rightarrow\ m^2-m(4r+1)+4r^2-2=0$

9. (d) : The given expansion is $(1+x)^m\left(1+\frac{1}{x}\right)^n$

$$=\frac{(1+x)^{m+n}}{x^n}$$

Thus the term independent of x in the given expansion is same as the co-efficient of x^n in $(1+x)^{m+n}$ which is ${}^{m+n}C_n=\frac{(m+n)!}{n!\,m!}=\frac{(m+n)!}{m!\,n!}$

10. (a) : The middle term in the expansion $(1+\alpha x)^4$ is the 3rd term

$\therefore\quad t_{2+1}={}^4C_2(\alpha x)^2={}^4C_2\alpha^2x^2$

and the same for $(1-\alpha x)^6$ is the 4th term.

$\therefore\quad t_{3+1}={}^6C_3(-\alpha x)^3={}^6C_3(-\alpha^3x^3)$

$\therefore$ By the problem

$$-{}^6C_3\alpha^3={}^4C_2\alpha^2\Rightarrow\frac{-6\times5\times4}{3!}\alpha=\frac{4\times3}{2!}$$

$$\Rightarrow\ -20\alpha=6\quad\therefore\quad\alpha=\frac{-6}{20}=\frac{-3}{10}$$

11. (a, d) : $t_{(r-1)+1}={}^{14}C_{r-1}\,x^{r-1}$

$t_{r+1}={}^{14}C_r\,x^r$

$t_{(r+1)+1}={}^{14}C_{r+1}x^{r-1}$

$\because$ r^{th} term, $(r+1)^{th}$ term and $(r+2)^{th}$ term are in A.P.

$\Rightarrow\ 14^2-14(4r+1)+4r^2-2=0$

[see the solution of Q. No. 8]

$\Rightarrow\ 4r^2-56r+180=0\ \Rightarrow\ r^2-14r+45=0$

$\Rightarrow\ (r-9)(r-5)=0\ \Rightarrow\ r=9$ or 5

12. (a, b, d) : $C_0-C_1+C_2-C_3+\ldots+(-1)^{m-1}\cdot C_{m-1}$

$={}^nC_0-{}^nC_1+{}^nC_2-{}^nC_3+\ldots+(-1)^{m-1}.\,{}^nC_{m-1}$

$={}^nC_0-({}^{n-1}C_0+{}^{n-1}C_1)+({}^{n-1}C_1+{}^{n-1}C_2)$

$-({}^{n-1}C_2+{}^{n-1}C_3)+\ldots.+(-1)^{m-1}({}^{n-1}C_{m-2}+{}^{n-1}C_{m-1})$

$=({}^nC_0-{}^{n-1}C_0)+(-1)^{m-1}\,.\,{}^{n-1}C_{m-1}$

$=0+(-1)^{m-1}\,.\,{}^{n-1}C_{m-1}$ (option (b))

$$=(-1)^{m-1}\cdot\frac{(n-1)(n-2)\ldots(n-m+1)}{(m-1)!}\text{ (option }(a))$$

Also from option (b) we get,

$C_0-C_1+C_2-C_3+\ldots+(-1)^{m-1}\cdot C_{m-1}$

$=(-1)^{m-1}\,.\,{}^{n-1}C_{m-1}$

$=(-1)^{m-1}\,.\,{}^{n-1}C_m$ (option (d))

$[\because\ {}^nC_r={}^nC_{n-r}]$

13. (a, c): The equation of the parabola is

$x^2+4y-6x+k=0$

$\Rightarrow\ (x-3)^2=-4y+9-k$

$\Rightarrow\ (x-3)^2=-4\left(y-\frac{9-k}{4}\right)$

Equation of directrix is $y-\frac{9-k}{4}=1\ \Rightarrow\ y=\frac{9-k}{4}+1$

$\therefore\ \frac{9-k}{4}+1=-1\ \Rightarrow\ k=17$

Vertex : $x-3=0\ \Rightarrow\ x=3$

and $y-\frac{9-17}{4}=0\ \Rightarrow\ y+2=0\ \Rightarrow\ y=-2$

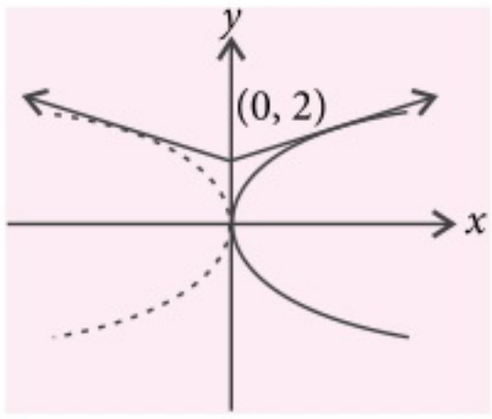
$\therefore$ vertex $\equiv(3,-2)$

Focus : $y+2=-1\ \Rightarrow\ y=-3$

$x-3=0\ \Rightarrow\ x=3$

$\therefore$ Focus $=(3,-3)$

14. (a, b) :



Any tangent is $y=mx+\frac{a}{m}$ ... (i)

Since (i) is passing through (0, 2)

$\therefore\quad 2=0+\frac{a}{m}\ \Rightarrow\ m=\frac{1}{2}$

Also, the y-axis is a tangent at the vertex of the parabola and slope of y-axis is ∞,

$\therefore\quad m=\infty,\ \frac{1}{2}$

Now, $\frac{3\pi}{4}=\frac{\pi}{2}\pm\tan^{-1}\frac{a}{2}\ \Rightarrow\ a=2,-2$

15. (b, d) : The slope of the normal to the ellipse at (α,β) is $\frac{-1}{-\frac{\alpha}{3\beta}}=\frac{3\beta}{\alpha}$

Since the normal is parallel to $6x-5y=2$

$\therefore\ \frac{6}{5}=\frac{3\beta}{\alpha}\ \Rightarrow\ 2\alpha=5\beta$ ... (i)

But (α,β) is point on the ellipse $x^2+3y^2=37$

i.e., $\alpha^2+3\beta^2=37$ ... (ii)

From (i) and (ii) we get $(5, 2), (-5, -2)$

16. (d) : The equation of the rectangular hyperbola is $xy=4^2$ ... (i)

and that of parabola is $y^2=4x$ ... (ii)

From (i) and (ii), we get

$$y^2 = 4 \cdot \frac{4^2}{y} \Rightarrow y^3 = 4^3$$

$\Rightarrow y^3 - 4^3 = 0 \Rightarrow (y-4)(y^2+4y+16) = 0$

$\Rightarrow y - 4 = 0 \quad \therefore \quad y = 4$

or, $y^2 + 4y + 16 = 0$

$$\therefore \quad y = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot 16}}{2 \cdot 1}$$

$$\Rightarrow \quad y = \frac{-4 \pm \sqrt{-48}}{2} \qquad \therefore \ y \notin R$$

Therefore when $y = 4$, then $x = 4$

Thus hyperbola intersects the parabola at (4, 4).

Again, when the hyperbola $\frac{x^2}{b^2} - \frac{y^2}{a^2} = 1$ passes through the focus of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then

$$\frac{(ae)^2}{b^2} = 1 \Rightarrow a^2e^2 = b^2 \quad \therefore \quad e^2 = \frac{b^2}{a^2}$$

$$\Rightarrow \quad e^2 = 1 - e^2 \Rightarrow 2e^2 = 1 \quad \therefore \quad e = \frac{1}{\sqrt{2}}$$

Now the eccentricity of the hyperbola

$$e' = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{1 + \frac{1}{e^2}} = \sqrt{1 + (\sqrt{2})^2} = \sqrt{3}$$

17. (c): Let the point $P(x, y)$

$\therefore \quad SP + S'P = 4$

$$\Rightarrow \sqrt{(x-\sqrt{3})^2 + y^2} + \sqrt{(x+\sqrt{3})^2 + y^2} = 4 \quad \ldots (1)$$

$$\Rightarrow (x+\sqrt{3})^2 + y^2 - (x-\sqrt{3})^2 - y^2 = 4\left\{\sqrt{(x+\sqrt{3})^2 + y^2} - \sqrt{(x-\sqrt{3})^2 + y^2}\right\}$$

$$\Rightarrow 4\sqrt{3}x = 4\left\{\sqrt{(x+\sqrt{3})^2 + y^2} - \sqrt{(x-\sqrt{3})^2 + y^2}\right\}$$

$$\Rightarrow \sqrt{(x+\sqrt{3})^2 + y^2} - \sqrt{(x-\sqrt{3})^2 + y^2} = \sqrt{3}x \quad \ldots (2)$$

Now adding (1) and (2), we get

$$2\sqrt{(x+i\sqrt{3})^2 + y^2} = 4 + \sqrt{3}x$$

or, $4(x^2 + y^2 + 2\sqrt{3}x + 3) = 16 + 8\sqrt{3}x + 3x^2$

or, $x^2 + 4y^2 = 4$ or, $\frac{x^2}{4} + y^2 = 1$

Again solving $2y - 1 = 0$ with $\frac{x^2}{4} + y^2 = 1$, we get $(\pm\sqrt{3}, 0)$. So the intercept $= 2\sqrt{3}$

18. (a) : The given equation of the parabola is

$x^2 + 2x + 12y - 11 = 0$

$\Rightarrow (x^2 + 2x + 1) = -12y + 12$

$\Rightarrow (x+1)^2 = -12(y-1)$

$\therefore$ Vertex = (–1, 1), Focus = (–1, –2), Directrix : $y = 4$

The circle on PQ as diameter touches $y = 4$ as PQ is focal chord.

19. (a) : The given expression is

$3^{2n+2} - 8n - 9 = 3^{2(n+1)} - 8n - 9$

$= 3^{2m} - 8(m-1) - 9$ [put $n + 1 = m$]

$= 3^{2m} - 8m + 8 - 9 = 9^m - 8m - 1$

$= (1+8)^m - 8m - 1$

$= (1 + {}^mC_1 \cdot 8 + {}^mC_2 \cdot 8^2 + {}^mC_3 \cdot 8^3 + \ldots + 8^m) - 8m - 1$

$= 8^2({}^mC_2 + {}^mC_3 \cdot 8 + \ldots\ldots 8^{m-2})$

Clearly, the given expression is divisible by 64 $\forall\, m \in N$

Also, we can prove that

$(1 + x)^n - nx - 1$ is always divisible by x^2.

20. (c): The given expansion is $\left(x + \frac{1}{x} + 2\right)^{25}$

$$= \left[(\sqrt{x})^2 + \left(\frac{1}{\sqrt{x}}\right)^2 + 2 \cdot \sqrt{x} \cdot \frac{1}{\sqrt{2}}\right]^{25} = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^{50}$$

Middle term of the above expansion is

$$t_{25+1}{}^{\text{th}} \text{ term} = {}^{50}C_{25}(\sqrt{x})^{25} \times \left(\frac{1}{\sqrt{x}}\right)^{25} = {}^{50}C_{25}$$

i.e., the middle term is independent of x.

So, Statement-1 is true.

But middle term of a binomial expansion is not always independent of the variable. So, Statement-2 is false.

21. (5) : $\frac{T_2}{T_3}$ in $(a+b)^n = \frac{{}^nC_1 a^{n-1} b}{{}^nC_2 a^{n-2} b^2} = \frac{2a}{(n-1)b}$

Again, $\frac{T_3}{T_4}$ in $(a+b)^n = \frac{{}^{n+3}C_2 a^{n+1} b^2}{{}^{n+3}C_3 a^n b^3} = \frac{3a}{(n+1)b}$

$\because \quad \frac{T_2}{T_3} = \frac{T_3}{T_4} \quad \therefore \quad \frac{2a}{(n-1)b} = \frac{3a}{(n+1)b} \Rightarrow n = 5$

22. (4) : $\lambda x^2 + 4xy + y^2 + \lambda x + 3y + 2 = 0$ will represent a parabola when $\lambda x^2 + 4xy + y^2$ is a perfect square.

Clearly, $\lambda x^2 + 4xy + y^2$ will be perfect square for $\lambda = 4$.

23. (9) : The given equation of the ellipse is

$$\frac{x^2}{9} + \frac{y^2}{25} = 1 \quad \therefore \quad a^2 = 25,\ b^2 = 9$$

$$\therefore \quad e = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$$

Major axis is along y-axis.

$$\therefore \quad \text{foci} \left(0, \pm 5 \cdot \frac{4}{5}\right) = (0, \pm 4)$$

Let the co-ordinate of a point on the ellipse be $(3\cos\theta, 5\sin\theta)$

Now equation of tangent at this point is

$$\frac{x \cdot 3\cos\theta}{9} + \frac{y \cdot 5\sin\theta}{25} = 1$$

$$\Rightarrow \quad \frac{x\cos\theta}{3} + \frac{y\sin\theta}{5} = 1$$

$$\Rightarrow \quad 5x\cos\theta + 3y\sin\theta = 15$$

$\therefore$ Product of perpendiculars is

$$= \frac{|3 \cdot 4\sin\theta - 15|}{\sqrt{(5\cos\theta)^2 + (3\sin\theta)^2}} \times \frac{|3(-4)\sin\theta - 15|}{\sqrt{(5\cos\theta)^2 + (3\sin\theta)^2}}$$

$$= \frac{(15 - 12\sin\theta)(15 + 12\sin\theta)}{25\cos^2\theta + 9\sin^2\theta}$$

$$= \frac{225 - 144\sin^2\theta}{25 - 16\sin^2\theta} = 9$$

24. (7) : Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \ (a > b)$$

Since the line, $\frac{x}{7} + \frac{y}{2} = 1$ intersects the ellipse at x-axis, *i.e.* at $(7, 0)$

$$\therefore \quad \frac{49}{a^2} = 1 \ \Rightarrow \ a^2 = 49$$

Again, $\frac{x}{3} - \frac{y}{5} = 1$ intersects the ellipse at y-axis *i.e.* at $(0, -5)$

$$\therefore \quad \frac{25}{b^2} = 1 \ \Rightarrow \ b^2 = 25$$

Now, $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{25}{49}} = \frac{\sqrt{24}}{7} = \frac{2\sqrt{6}}{7}$ $\therefore \lambda = 7$

25. (4) : Let the equation of hyperbola and the circle are

$$xy = k^2 \qquad \ldots (1)$$

and $x^2 + y^2 + 2gx + 2fy + c = 0$ ... (2)

where $g^2 + f^2 - c = r^2$ (given) ... (3)

Eliminating y between (1) and (2), we get a biquadratic in x giving us the abscissas of the four points of intersection as

$$x^4 + 2gx^3 + cx^2 + 2fk^2x + k^4 = 0$$

Its roots are x_1, x_2, x_3 and x_4, then

$$\Sigma x_1 = -2g, \Sigma x_1x_2 = c$$

$$\therefore \quad x_1^2 + x_2^2 + x_3^2 + x_4^2 = (\Sigma x_1)^2 - 2\Sigma x_1x_2$$

$$= 4g^2 - 2c$$

Similarly, $y_1^2 + y_2^2 + y_3^2 + y_4^2 = 4f^2 - 2c$

$$\therefore \quad CP^2 + CQ^2 + CR^2 + CS^2$$

$$= (x_1^2 + y_1^2) + (x_2^2 + y_2^2) + (x_3^2 + y_3^2) + (x_4^2 + y_4^2)$$

$$= \Sigma x_1^2 + \Sigma y_1^2 = 4g^2 - 2c + 4f^2 - 2c$$

$$= 4(g^2 + f^2 - c) = 4r^2$$

PAPER-II

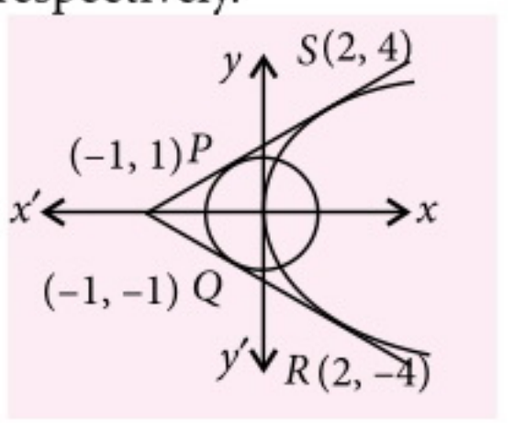
1. (d) : Let $y = mx + \frac{2}{m}$ be a tangent to $y^2 = 8x$.

The above straight line will also be the tangent to the circle $x^2 + y^2 = 2$.

$$\left|\frac{\frac{2}{m}}{\sqrt{1 + m^2}}\right| = \sqrt{2} \ \Rightarrow \ \frac{4}{m^2} \cdot \frac{1}{1 + m^2} = 2$$

$$\Rightarrow \quad m^2(1 + m^2) = 2 \ \Rightarrow \ m^2 = 1 \ \therefore \ m = \pm 1$$

The points of contact being $(-1, 1)$, $(2, 4)$ $(2, -4)$ and $(-1, -1)$ respectively.



The area of quadrilateral

$$PQRS = \left[\frac{1}{2}(1 + 4) \times 3\right] \times 2 = 15$$

2. (a) : Let the point be (α, β)

$\therefore \ \beta^2 = 64\,\alpha$

If the distance of the point from the straight line $4x + 3y + 35$ is d, then

$$d=\frac{4\alpha+3\beta+35}{\pm\sqrt{4^2+3^2}}=\frac{\frac{\beta^2}{16}+3\beta+35}{5}$$

[$\because$ numerator is +ve]

$$=\frac{\beta^2+48\beta+560}{80}=\frac{(\beta+24)^2-4^2}{80}$$

$\therefore$ d will be minimum when $\beta + 24 = 0$ *i.e.*, $\beta = -24$

$$\therefore \alpha=\frac{576}{64}=9$$

$\therefore$ The required nearest point is (9, –24).

3. (a) : $y=mx+\frac{2\sqrt{3}}{m}$ is a tangent to the parabola $y^2=8\sqrt{3}x$.

The above straight line will be the tangent to the hyperbola $4x^2 - y^2 = 4$ if

$$\frac{2\sqrt{3}}{m}=\sqrt{1\cdot m^2-4} \Rightarrow \frac{12}{m^2}=(m^2-4)$$

$\Rightarrow$ $m^2 (m^2 - 4) = 12 \Rightarrow (m^2)^2 - 4m^2 - 12 = 0$

$\Rightarrow$ $(m^2)^2 - 6 m^2 + 2m^2 - 12 = 0$

$\Rightarrow$ $m^2(m^2 - 6) + 2(m^2 - 6) = 0$

$\Rightarrow$ $(m^2 + 2)(m^2 - 6) = 0 \Rightarrow m=\pm\sqrt{6}$ [$\because m \in R$]

For positive slope, $m=\sqrt{6}$

$\therefore$ required tangent is $y=\sqrt{6}x+\sqrt{2}$.

4. (c): The given equation of the circle is

$$\frac{x^2}{144}+\frac{y^2}{25}=1 \therefore e=\sqrt{1-\frac{25}{144}}=\sqrt{\frac{119}{144}}$$

$\therefore$ Foci : $(\pm\sqrt{119},0)$

Centre of the circle is $(0,\sqrt{2})$

$\text{radius}=\sqrt{119+2}=\sqrt{121}=11$

5. (b) : The given equation of the parabola is

$y^2 = 12x = 4 \cdot 3x$

$\therefore$ any point on the parabola will be $(3t^2, 6t)$

$$\text{Now, } 2y\frac{dy}{dx}=12\Rightarrow\frac{dy}{dx}=\frac{6}{y}$$

$$\therefore \left[\frac{dy}{dx}\right]_{(3t^2,6t)}=\frac{6}{6t}=\frac{1}{t}$$

$\therefore$ by the condition given, we have

$$\frac{1}{t}\times 4=-1 \Rightarrow t=-4$$

$\therefore$ the point on the parabola is (48, –24)

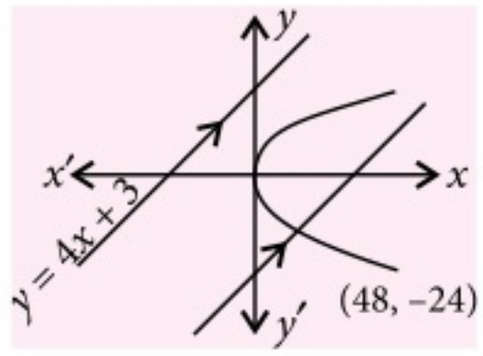
$\therefore$ equation of the normal is $y + 24 = 4 \times (x - 48)$

$\Rightarrow$ $y = 4x - 216$

$\therefore$ distance between the parallel lines $y = 4x + 3$ and $y = 4x - 216$

$$=\frac{3-(-216)}{\sqrt{4^2+(-1)^2}}$$

$$=\frac{219}{\sqrt{17}}$$



6. (d) : The given equation of the ellipses are

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \text{... (i)}$$

and $$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1 \quad \text{... (ii)}$$

The equation of the graph passing through the intersection of (i) and (ii) is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-1+\lambda\left(\frac{x^2}{b^2}+\frac{y^2}{a^2}-1\right)=0$$

$$\Rightarrow x^2\left(\frac{1}{a^2}+\frac{1}{b^2}\lambda\right)+y^2\left(\frac{1}{b^2}+\frac{1}{a^2}\cdot\lambda\right)-1-\lambda=0$$

...(iii)

The equation (iii) will be the equation of a circle if

$$\frac{1}{a^2}+\frac{\lambda}{b^2}=\frac{1}{b^2}+\frac{\lambda}{a^2} \Rightarrow \lambda\left(\frac{1}{b^2}-\frac{1}{a^2}\right)=\frac{1}{b^2}-\frac{1}{a^2}$$

$\Rightarrow$ $\lambda = 1$

$\therefore$ The equation of the circle is

$$\left(\frac{1}{a^2}+\frac{1}{b^2}\right)x^2+\left(\frac{1}{b^2}+\frac{1}{a^2}\right)y^2=1+1=2$$

$$\Rightarrow x^2+y^2=\frac{2a^2b^2}{a^2+b^2}$$

7. (d) : Since, $(1 + 3x + 2x^2)^6$

$= 1 + {}^6C_1(3x + 2x^2) + {}^6C_2(3x + 2x^2)^2 + {}^6C_3(3x + 2x^2)^3$
$+ {}^6C_4(3x + 2x^2)^4 + {}^6C_5(3x + 2x^2)^5 + (3x + 2x^2)^6$

$\therefore$ the co-efficient of x^{11} in $(1 + 3x + 2x^2)^6$

= The co-efficient of x^{11} in $(3x + 2x^2)^6$

$= {}^6C_5 \cdot 3 \cdot 2^5 = 6 \times 3 \times 32 = 576$

8. (a) : The given expression is

$a - {}^nC_1(a - 1) + {}^nC_2(a - 2) + ... + (-1)^n (a - n)$

$= a[1 - {}^nC_1 + {}^nC_2 - ... + (-1)^n] + nC_1 - 2\cdot{}^nC_2 + 3{}^nC_3 -$
$.....(-1)^n (-n)$

Now ${}^nC_1 - 2 \cdot {}^nC_2 + 3 \cdot {}^nC_3 - \ldots (-n) \cdot (-1)^n$

$$= n - 2 \cdot \frac{n(n-1)}{2!} + 3\frac{n(n-1)(n-2)}{3!} \ldots + n(-1)^{n-1}$$

$$= n\left[1 - (n-1) + \frac{(n-1)(n-2)}{2!} - \ldots + (-1)^{n-1}\right]$$

$= n(1 - 1)^{n-1} = 0$

Again $(1 + x)^n = 1 + {}^nC_1 x + {}^nC_2 x^2 + \ldots + x^n$

Putting $x = -1$ on both sides, we get

$1 - {}^nC_1 + {}^nC_2 - \ldots + (-1)^n = (1 - 1)^n = 0$

$\therefore$ The given expression becomes

$a - {}^nC_1(a - 1) + {}^nC_2(a - 2) + \ldots + (-1)^n (a - n)$

$= a[0] + n\,[1 - {}^{n-1}C_1 + {}^{n-1}C_2 - \ldots + (-1)^{n-1}]$

$$= 0 + n \times \left[1 - (n-1) + \frac{(n-1)(n-2)}{2!} + \ldots + (-1)^{n-1}\right]$$

$= 0 + n \times 0 = 0$

9. (a) : $(\sqrt{2} + 3^{1/5})^{10}$

Clearly the first term and 11^{th} term are rational.

$\therefore$ the sum of the rational terms in the given binomial $= (\sqrt{2})^{10} + (3^{1/5})^{10} = 41$

10. (d) : The given expression is

$(1 + x^2)^{12}\,(1 + x^{12})\,(1 + x^{24})$

$= [1 + {}^{12}C_1 x^2 + \ldots + {}^{12}C_6 x^{12} + \ldots + x^{24}]$

$\times (1 + x^{12} + x^{24} + x^{36})$

$\therefore$ the co-efficient of x^{24} is $1 + {}^{12}C_6 + 1 = {}^{12}C_6 + 2$

11. (a, c): The given equation of the parabola is $Ay^2 + By + Cx + D = 0$

$$\Rightarrow Ay^2 + By = -Cx - D \Rightarrow y^2 + \frac{B}{A}y = -\frac{Cx}{A} - \frac{D}{A}$$

$$\Rightarrow (y)^2 + 2 \cdot y \cdot \frac{B}{2A} + \left(\frac{B}{2A}\right)^2 = -\frac{Cx}{A} - \frac{D}{A} + \frac{B^2}{4A^2}$$

$$\Rightarrow \left(y + \frac{B}{2A}\right)^2 = -\frac{Cx}{A} + \frac{B^2 - 4AD}{4A^2}$$

$$\Rightarrow \left(y + \frac{B}{2A}\right)^2 = -\frac{C}{A}\left(x - \frac{B^2 - 4AD}{4AC}\right)$$

- Length of latus rectum $\left|\frac{C}{A}\right|$
- axis $y + \frac{B}{2A} = 0 \Rightarrow y = -\frac{B}{2A}$ *i.e*, axis is parallel to x-axis.
- Focus : $(-a, 0)$

$\therefore$ ordinate of focus $y + \frac{B}{2A} = 0$ *i.e*, $y = -\frac{B}{2A}$

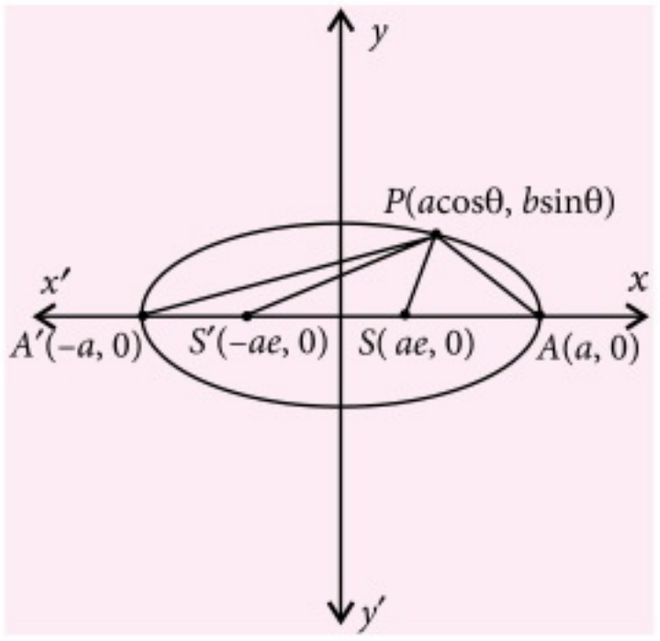
- abscissa of focus will be obtained by

$$x - \frac{B^2 - 4AD}{4AC} = -\frac{C}{4A} \Rightarrow x = \frac{B^2 - 4AD}{4AC} - \frac{C}{4A}$$

$$= \frac{B^2 - 4AD - C^2}{4AC}$$

12. (b,c) : Let $P(a\cos\theta, b\sin\theta)$ be a point on $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\therefore \frac{\Delta AA'P}{\Delta SS'P} = \frac{ab\sin\theta}{abe\sin\theta} = \frac{1}{e}$$



- $K > 1$ as $e < 1$
- K is only dependable on e.

13. (a,c,d) : The given equation of hyperbola is $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1\left(0 < \alpha < \frac{\pi}{2}\right)$

- Eccentricity

$$= \sqrt{1 + \frac{\sin^2\alpha}{\cos^2\alpha}} = \sqrt{1 + \tan^2\alpha} = \sec\alpha$$

- Foci : $(\pm ae, 0) = (\pm \cos\alpha \cdot \sec\alpha, 0) = (\pm 1, 0)$
- Directrix : $x = \pm\frac{a}{e} = \pm\frac{\cos\alpha}{\sec\alpha} = \pm\cos^2\alpha$
- Vertex : $(\pm a, 0) = (\pm \cos\alpha, 0)$

14. (a, d) : The given expression is $\left(\frac{x}{2} + 2\right)^8$

t_{4+1} = middle term $= {}^8C_4\left(\frac{x}{2}\right)^4 (2)^4 = {}^8C_4 x^4$

$\therefore$ ${}^8C_4 x^4 = 1120$

$x^4 = 16 = (\pm 2)^4 \Rightarrow \quad x = \pm 2$

15. (a,b,c) : $(1 + bx^2 + cx^3)^n$
$= 1 + {}^nC_1(bx^2 + cx^3) + {}^nC_2(bx^2 + cx^3)^2 +$ (terms having power 6 or more)
$\therefore \quad a_2 =$ Co-efficient of $x^2 = {}^nC_1 \cdot b = nb$

and co-efficient of $x^4 = a_4 = {}^nC_2 b^2 = \frac{n(n-1)}{2}\left(\frac{a_2}{n}\right)^2$

$$\Rightarrow 2na_4 = (n-1)a_2^2 =) \frac{2a_4}{a_2^2} = \frac{n-1}{n}$$

$$a_2 + a_4 = n \cdot b + \frac{n(n-1)}{2} b^2 = nb\left(1 + \frac{(n-1)b}{2}\right)$$

16. (c): Given $(\sqrt{2} + \sqrt[5]{3})^{120}$

$$\therefore T_{r+1} = {}^{120}C_r \left(2^{\frac{1}{2}}\right)^{120-r} \cdot \left(3^{\frac{1}{5}}\right)^r = {}^{120}C_r 2^{60-\frac{r}{2}} \cdot 3^{\frac{r}{5}}$$

For rational terms $\frac{r}{2}, \frac{r}{5}$ are integers

$\therefore r =$ the multiple of the L.C.M of $(2, 5) = 10$
As, $0 \le r \le 120$
$\therefore r = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 120$
$\therefore$ number of rational terms $= 13$
$\therefore$ number of irrational terms $= 121 - 13 = 108$

17. (b) : The given expansion $(2^{1/5} + 7^{1/7})^{105}$

$$\therefore T_{r+1} = {}^{105}C_r (2^{1/5})^{105-r} \cdot (7^{1/7})^r$$
$$= {}^{105}C_r 2^{21-\frac{r}{5}} \cdot 7^{\frac{r}{7}}$$

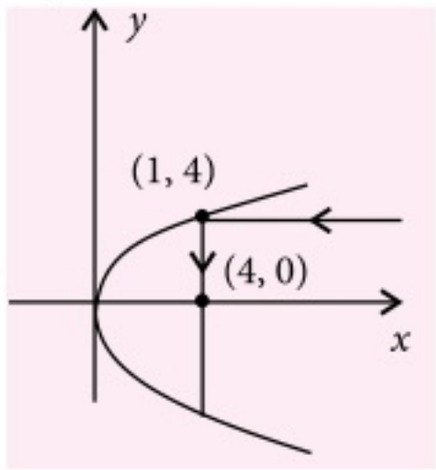
For rational terms r is the multiple of $7 \times 5 = 35$
$\therefore \quad 0 \le r \le 105$
$\therefore \quad r = 0, 35, 70, 105$
So, sum of rational terms is $T_1 + T_{36} + T_{71} + T_{106}$
$= {}^{105}C_0\, 2^{21} \cdot 7^0 + {}^{105}C_{35}\, 2^{14}\, 7^5 + {}^{105}C_{70}\, 2^7 \cdot 7^{10} + {}^{105}C_{105} + 2^0 \cdot 7^{15}$

18. (a) : Given $(7^{1/3} + 11^{1/9})^{5832}$

$$T_{r+1} = {}^{5832}C_r 7^{\frac{5832-r}{3}} \cdot 11^{\frac{r}{9}}$$

Now T_{r+1} will be rational if $\frac{r}{9}, \frac{r}{3}$ are integers
$\therefore \quad r$ is a multiple of 9
$\therefore \quad 0 \le r \le 5832$ and $0 \le \frac{r}{9} \le 648$
$\therefore \quad \frac{r}{9} = 0, 1, 2, 3, \ldots, 648$
$\therefore$ The total number of terms free from radicals
$= 648 + 1 = 649.$

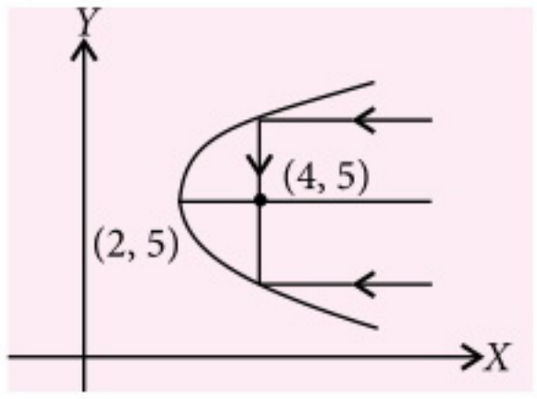
19. (b) : Given $y^2 = 16x$
Focus $(4, 0)$ and the point of intersection of $y = 4$ and the parabola $y^2 = 16x$ is $(1, 4)$



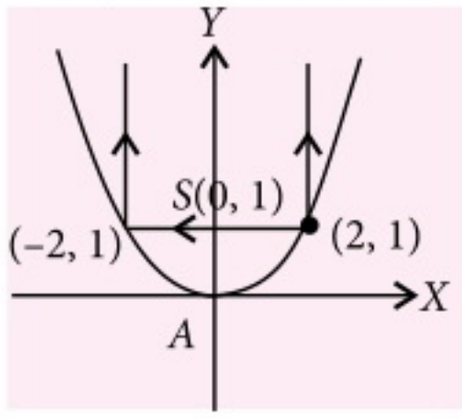
$\therefore$ equation of the reflected ray is given by

$$y - 0 = \frac{4-0}{1-4}(x-4) \Rightarrow 3y + 4x = 16$$

20. (a) : The given equation of the parabola is
$y^2 - 10y - 8x + 41 = 0 \Rightarrow (y-5)^2 = 8(x-2)$
$\Rightarrow Y^2 = 8X$
where $Y = y - 5$ and $X = x - 2$
$\Rightarrow$ Focus $(a, 0) = (2, 0)$
$\Rightarrow X = 2 \Rightarrow x - 2 = 2 \Rightarrow x = 4$
and $Y = 0 \Rightarrow y - 5 = 0 \Rightarrow y = 5$
$\therefore$ After reflection, the ray must pass through the focus $(4, 5)$



21. (c): The given equation of parabola is $x^2 = 4y$
$\therefore$ Focus $(0, 1)$ and the point of intersection of $x = 2$ and parabola $x^2 = 4y$ is $(2, 1)$
Now equation of the first reflection is $y = 1$ which is perpendicular to the incident ray, hence the second reflected ray is parallel to incident ray and pass through the point $(-2, 1)$ is $x = -2$ *i.e.*, $x + 2 = 0$.



22. A → p, s; B → p, q, r; C → p, q; D → p

23. A → r; B → s; C → p; D → q

MATHS MUSING

SOLUTION SET-141

1. **(d) :** $ax^3 + bx^2 + c = (x^2 + mx - 1)(ax - c)$

$\therefore\ b = am - c,\ a + cm = 0$. Eliminating m

$a^2 + c(b + c) = 0.$

2. **(c) :** 10 lines intersect at $\binom{10}{2} = 45$ points.

The number of lines formed by joining these points is $\binom{45}{2} = 990$. It includes lines formed by points on the old lines.

The number of new lines $= 990 - 10\binom{9}{2}$

$= 990 - 360 = 630.$

3. **(c) :** $\cos^8 x = \frac{1}{2^8}(e^{ix} + e^{-ix})^8$

$= \frac{1}{2^7}(\cos 8x + 8\cos 6x + \ldots)$

$I = \frac{1}{16}\int_0^{\pi} \cos^2 6x\, dx = \frac{\pi}{32}$

4. **(d) :** S. D. between and $\vec{r} = \vec{a} + s\vec{b}$ and $\vec{r} = \vec{c} + t\vec{d}$ is $|(\vec{c} - \vec{a}) \cdot \vec{b} \times \vec{d}| \div |\vec{b} \times \vec{d}|$

$\vec{a} = -(\hat{i} + \hat{j} + \hat{k}),\ \vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{c} = -\hat{i},\ \vec{d} = 3\hat{i} + 4\hat{i} + 5\hat{i} \Rightarrow$ S. D $= \frac{1}{\sqrt{6}}$

5. **(a):** 6621, 6341, 6322, 4811, 9421, 9221, 3382, 3342. Taking the permutations

$N = 2(4!) + 5\left(\frac{4!}{2!}\right) + \frac{4!}{3!} = 112$

6. **(b),(d):** $13^{99} = (4 + 9)^{99} = 4^{99} \bmod 9^2$

$= 2^{198} \bmod 81 = 8^{66} \bmod 81 = (1 - 9)^{66} \bmod 81$

$= 1 - 9 \cdot (6 \bmod 8) = 1 - 9(72 - 6) = 55 \bmod 81$

Remainder in $55 = 5 \times 11$

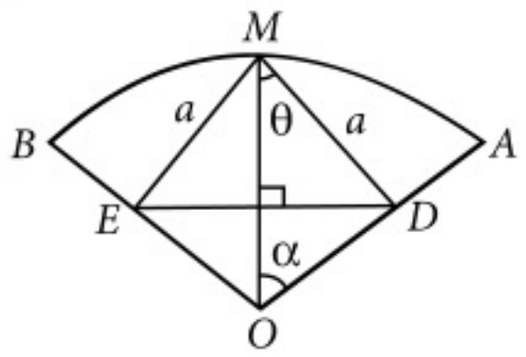
7. **(c) :** $\alpha = \frac{\pi}{3}, \theta = \frac{\pi}{6}$

$a(\cos\theta + \sin\theta\ \cot\alpha) = 1$

$\Rightarrow\ a = \frac{\sin\alpha}{\sin(\theta + \alpha)} = \frac{\sqrt{3}}{2}$

$\Delta = \frac{\sqrt{3}}{4}\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3\sqrt{3}}{16}$

8. **(a):**



$\Delta = \frac{1}{2}a^2 \sin 2\theta = \frac{\sin\theta\cos\theta}{\sin^2\left(\theta + \frac{\pi}{3}\right)}\cdot\frac{3}{4}$

Differentiation $\Rightarrow \theta = \alpha = \frac{\pi}{3}$

$\Delta_{max} = \frac{\sqrt{3}}{4}$

9. (4) : $\binom{n}{6} = 6\binom{n}{3} \Rightarrow (n-3)(n-4)(n-5) = 720$

$= 10 \cdot 9 \cdot 8 \Rightarrow n = 13$

10. (c): **P.** $\binom{9}{5} = \binom{9}{4} = 126$, digit sum 9

Q. $\binom{9-1}{5-1} = \binom{8}{4} = 70$, digit sum 7

R. $5!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right) = 44,$

digit sum 8

S. $\binom{9+5-1}{9} = \binom{13}{9} = \binom{13}{4} = 715,$

digit sum 13.

Solution Sender of Maths Musing

SET-141

1. Dusmant Sahoo (Bhubaneswar, Odisha)
2. Biranchi Narayan Patra (Bihar)

YOU ASKED WE ANSWERED

Do you have a question that you just can't get answered?

Use the vast expertise of our mtg team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.

The best questions and their solutions will be printed in this column each month.

Q1. Find the intervals in which the function f given by $f(x) = \sin x + \cos x$, $0 \le x \le 2\pi$ is strictly increasing or strictly decreasing.

[Krishna Kumar; Patna]

Ans. We have

$$f(x) = \sin x + \cos x$$

or $\quad f'(x) = \cos x - \sin x$

Now $f'(x) = 0$ gives $\sin x = \cos x$ which gives that $x = \frac{\pi}{4}$ or $\frac{5\pi}{4}$ as $0 \le x \le 2\pi$.

The numbers $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$ divide the interval $[0, 2\pi]$ into three disjoint intervals, namely $\left[\left(0, \frac{\pi}{4}\right), \left(\frac{\pi}{4}, \frac{5\pi}{4}\right) \text{ and } \left(\frac{5\pi}{4}, 2\pi\right)\right]$.

Interval	Sign of $f'(x)$	Nature of Function
$\left(0, \frac{\pi}{4}\right)$	> 0	f is strictly increasing
$\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$	< 0	f is strictly decreasing
$\left(\frac{5\pi}{4}, 2\pi\right)$	> 0	f is strictly increasing

Q2. Solve the equation

$$\tan^{-1}\sqrt{x^2+x} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$$

[Subhdara; Bangalore]

Ans. Given equation exist, if

$$x^2 + x \ge 0 \quad \text{and} \quad 0 < \sqrt{x^2+x+1} \le 1$$

$[\because\ x^2 + x + 1$ is always greater than zero$]$

Now,

$$x^2 + x \ge 0 \quad \text{and} \quad x^2 + x + 1 \le 1$$

$$\Rightarrow \quad x^2 + x \ge 0 \quad \text{and} \quad x^2 + x \le 0$$

$$\Rightarrow \quad x^2 + x = 0 \quad i.e., \quad x(x+1) = 0$$

Hence $x = 0$ and -1 are the solutions of the given equation.

Q3. Prove that

If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, then $x + y + z = xyz$ & If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$, then $xy + yz + zx = 1$.

[Saurabh B. Kasote; Kolhapur]

Ans. $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$

$$\tan^{-1}\left(\frac{x+y}{1-xy}\right) + \tan^{-1}z = \pi$$

$$\tan^{-1}\left(\frac{\left(\frac{x+y}{1-xy} + z\right)}{1 - \left(\frac{x+y}{1-xy}\right)(z)}\right) = \pi$$

$$\frac{x+y+z(1-xy)}{(1-xy)-z(x+y)} = \tan\pi$$

$$\frac{x+y+z-xyz}{1-xy-zx-zy} = 0$$

$$x + y + z = xyz$$

Hence proved.

$$\tan^{-1}x + \tan^{-1}z = \frac{\pi}{2} - \tan^{-1}z$$

$$\tan(\tan^{-1}x + \tan^{-1}y) = \tan\left(\frac{\pi}{z} - \tan^{-1}z\right)$$

$$\frac{x+y}{1-xy} = \cot(\tan^{-1}z)$$

$$\frac{x+y}{1-xy} = \cot\left(\cot^{-1}\left(\frac{1}{2}\right)\right)$$

$$\Rightarrow \quad \frac{x+y}{1-xy} = \frac{1}{z}$$

$$zx + zy = 1 - xy$$

$$\Rightarrow \quad xy + yz + zx = 1$$

Hence proved.

PRACTICE PAPER 2015

JEE (Main & Advanced) and other PETs

PAPER-I

SECTION-I

MULTIPLE CORRECT CHOICE TYPE

This section contains 10 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which ONE or MORE may be correct.

1. A curve that passing through (2, 4) and having subnormal of constant length of 8 units can be

(a) $y^2 = 16x - 16$ (b) $y^2 = -16x + 48$

(c) $x^2 = 16y - 60$ (d) $x^2 = -16y + 68$

2. If the conjugate of $(x + iy)(1 - 2i)$ be $1 + i$, then

(a) $x = \frac{1}{5}$ (b) $x + iy = \frac{1}{5}(3 + i)$

(c) $x - iy = \frac{1}{5}(3 + i)$ (d) $x + iy = \frac{1 - i}{1 - 2i}$

3. If $a + b + c = 0$, then the roots of the equation $(b + c - a)x^2 + (c + a - b)x + (a + b - c) = 0$ can be

(a) imaginary (b) real and equal

(c) real and unequal (d) none of these

4. The solution of $\left(\frac{dy}{dx}\right)^2 + 2y \cot x \frac{dy}{dx} = y^2$ is

(a) $y - \frac{c}{1 + \cos x} = 0$ (b) $y = \frac{c}{1 - \cos x}$

(c) $x = 2\sin^{-1}\left(\sqrt{\frac{c}{y}}\right)$ (d) none of these

5. $\sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{k=1}^{j} 1$ is equal to

(a) $\frac{n(n+1)(n+2)}{6}$ (b) Σn^2

(c) ${}^nC_3$ (d) ${}^{n+2}C_3$

6. If A and B are two matrices such that $AB = BA$, then $\forall\, n \in N$

(a) $A^nB = BA^n$ (b) $(AB)^n = A^nB^n$

(c) $(A + B)^n = {}^nC_0 A^n + {}^nC_1 A^{n-1} B + {}^nC_2 A^{n-2} B^2 + \ldots + {}^nC_n B^n$

(d) $A^{2n} - B^{2n} = (A^n - B^n)(A^n + B^n)$

7. If the graph of the function $f(x)$ is symmetrical about two lines $x = a$ and $x = b$ then $f(x)$ must be periodic with period

(a) $\frac{b-a}{2}$ (b) $b - a$

(c) $2(b - a)$ (d) none of these

8. The function $f(x) = \begin{cases} |x-3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$ is

(a) continuous at $x = 1$

(b) differentiable at $x = 1$

(c) continuous at $x = 3$

(d) differentiable at $x = 3$

9. Let $f(x) = \sin(\pi x) - 4x(1 - x)$, then

(a) $\frac{\sin(\pi x)}{x(1-x)} \leq 4\ \forall\ x \in (0, 1)$

(b) $f'\left(\frac{5}{8}\right) + f'\left(\frac{3}{8}\right) = 0$

(c) $f''(x) = 0$ has at least two solutions in $(0, 1)$

(d) Rolle's theorem can be applied to $f'(x)$ in $[\alpha, 1/2]$ for some $\alpha \in (0, 1/2)$

10. Let $\int \sqrt{\frac{x}{1-x^3}}\, dx = \frac{2}{3} g(f(x)) + c$. Then

(a) $g(x) = \cot^{-1}(x)$ and $f(x) = \sqrt{\frac{1-x^3}{x^3}}$

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(b) $g(x)=\tan^{-1}(x)$ and $f(x)=\sqrt{\dfrac{1-x^3}{x^3}}$

(c) $g(x)=\cot^{-1}(x)$ and $f(x)=\sqrt{\dfrac{x^3}{1-x^3}}$

(d) $g(x)=\tan^{-1}(x)$ and $f(x)=\sqrt{\dfrac{x^3}{1-x^3}}$

SECTION-II

ONE INTEGER VALUE CORRECT TYPE

This section contains 10 questions. Each question, when worked out will result in one integer from 0 to 9 (both inclusive).

11. If α, β, γ are the roots of the equation $x^3 - 9x^2 + 14x + 24 = 0$, then find the numerical value of $|\alpha + \beta + \gamma + \alpha\beta + \beta\gamma + \gamma\alpha + \alpha\beta\gamma|$.

12. The least degree of a polynomial with integer coefficient whose one of the roots may be $\cos 12^\circ$ is

13. If $\sin^{-1}x + \sin^{-1}y = \pi$ and, if $x = \lambda y$, then the value of $39^{2\lambda} + 5^{\lambda} - 1525$ must be

14. The number of solution/s of the equation

$$\int_{-1}^{x}\left(8t^2+\frac{28t}{3}+4\right)dt=\frac{\frac{3}{2}x+1}{\log_{x+1}\sqrt{x+1}}$$ is

15. In a ΔABC, if a is the arithmetic mean and b, c are two geometric means between any two positive numbers. Then $\dfrac{\sin^3 B+\sin^3 C}{\sin A\sin B\sin C}$ is equal to

16. Let 'm' denotes the number of ways in which 4 different balls of green colour and 4 different balls of red colour can be distributed equally among 4 persons if each person has balls of the same colour and 'n' be corresponding figure when all the four persons have balls of different colour. Find $\dfrac{(m+n)}{132}$.

17. Let $f(x) = [3 + 4\sin x]$ (where [ ] denotes the greatest integer function). If sum of all the values of 'x' in $[\pi, 2\pi]$ where $f(x)$ fails to be differentiable, is $\dfrac{k\pi}{2}$, then the value of $\dfrac{k}{8}$, is

18. The polynomial $p(x) = 1 - x + x^2 - x^3 + \ldots + x^{16} - x^{17}$ can be written as a polynomial in y where $y = x + 1$, then let coefficient of y^2 be k then $\left[\dfrac{k}{400}\right]$ (where [.] denote greatest integer function) is _____

19. Let A be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. The tangents at the points $B(1, 7)$ and $C(4, -2)$ on the circle meet at the point D. If Δ denotes the area of the quadrilateral $ABCD$, then $\sqrt{\dfrac{\Delta}{3}}$ is equal to

20. Find the integer n for which $\lim\limits_{x\to 0}\dfrac{(\cos x-1)(\cos x-e^x)}{x^n}$ is a finite non-zero number.

PAPER-II

SECTION-I

SINGLE CORRECT OPTION

This section contains 10 multiple choice questions. Each question has four choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

1. If x and y are positive real numbers and m, n are any positive integers and $E=\dfrac{x^n y^m}{(1+x^{2n})(1+y^{2m})}$ then

(a) $E>\dfrac{1}{4}$ (b) $E>\dfrac{1}{2}$

(c) $E\le\dfrac{1}{4}$ (d) $E<\dfrac{1}{8}$

2. Four persons are selected at random out of 3 men, 2 women and 4 children. What is the chance that exactly 2 of them are children?

(a) $\dfrac{9}{21}$ (b) $\dfrac{10}{23}$ (c) $\dfrac{11}{24}$ (d) $\dfrac{10}{21}$

3. For the equation $x^2 - (k + 1)x + (k^2 + k - 8) = 0$ if one root is greater than 2 and other is less than 2, then k can take any value between

(a) $(-2, 3)$ (b) $(-11/3, 3)$

(c) $(-11/3, -2)$ (d) $(0, 5)$

4. If z is a complex number satisfying $|z+1-i| \le 1$, then the maximum value of $|z|$ is

(a) $\sqrt{2}$ (b) $\sqrt{2}-1$

(c) $\sqrt{2}+1$ (d) 1

5. The value of $\int_1^{e^{37}} \frac{\pi \sin(\pi \ln x)}{x} dx$ is

(a) 1 (b) –1

(c) 2 (d) none of these

6. For a parabola having focus at S, vertex at A such that $SA = l_1$ units and focal chord PQ of length l_2 units is given, then ar(ΔAPQ) is

(a) $l_1 l_2$ (b) $l_1\sqrt{l_1 l_2}$

(c) $l_2\sqrt{l_1 l_2}$ (d) $\frac{l_2^3}{2l_1}$

7. If $\cos^4\theta \sec^2\alpha$, 1/2 and $\sin^4\theta \operatorname{cosec}^2\alpha$ are in A.P. then $\cos^8\theta \sec^6\alpha$, 1/2 and $\sin^8\theta \operatorname{cosec}^6\alpha$ are in

(a) A.P. (b) G.P.

(c) H.P. (d) none of these

8. The point of intersection of the plane $\vec{r}\cdot 3\hat{i} - 5\hat{j} + 2\hat{k} = 6$ with the straight line passing through the origin and perpendicular to the plane $2x - y - z = 4$ is

(a) (1, –1, –1) (b) (–1, –1, 2)

(c) (4, 2, 2) (d) $\left(\frac{4}{3}, \frac{-2}{3}, \frac{-2}{3}\right)$

9. The function $f(x) = \log_e(1+x) - \frac{2x}{2+x}$ is increasing on

(a) $(-1, \infty)$ (b) $(-\infty, 0)$

(c) $(-\infty, \infty)$ (d) none of these

10. If $\lim_{x\to\infty} f(x^2) = a$ (a finite number), then which of the following is/are true?

(a) $\lim_{x\to\infty} x^2 f'(x^2) = 0$

(b) $\lim_{x\to\infty} x^2 f'(x^2) = 2a$

(c) $\lim_{x\to\infty} x^4 f''(x^2) = 0$

(d) both (a) and (c)

SECTION-II

PARAGRAPH TYPE

This section contains 3 paragraph. Based upon each of the paragraphs 2 multiple choice questions have to be answered. Each of these questions has four choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

Paragraph for Q. No. 11 and 12

Suppose equation is $f(x) - g(x) = 0$ or $f(x) = g(x) = y$ say, then draw the graphs of $y = f(x)$ and $y = g(x)$. If graphs of $y = f(x)$ and $y = g(x)$ cuts at one, two, three, …, no points, then number of solutions are one, two, three, …, zero respectively.

11. The number of solutions of $\sin x = \frac{|x|}{10}$ is

(a) 4 (b) 6

(c) 8 (d) None of these

12. Total number of solutions of the equation $3x + 2\tan x = \frac{5\pi}{2}$ in $x \in [0, 2\pi]$ is equal to

(a) 1 (b) 2 (c) 3 (d) 4

Paragraph for Q. No. 13 and 14

A and B are two matrices of same order 3×3, where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 5 & 6 & 8 \end{pmatrix} \text{ and } B = \begin{pmatrix} 3 & 2 & 5 \\ 2 & 3 & 8 \\ 7 & 2 & 9 \end{pmatrix}$$

13. The value of adj (adj A) is equal to

(a) $2A$ (b) $4A$

(c) 8A (d) none of these

14. The value of |adj (adj A)| is equal to

(a) 9 (b) 16 (c) 25 (d) 1

Paragraph for Q. No. 15 and 16

Consider two functions $f(x) = \lim_{n\to\infty}\left(\cos\frac{x}{\sqrt{n}}\right)^n$ and $g(x) = -x^{4b}$, where

$$b = \lim_{x\to\infty} \sqrt{x^2+x+1} - \sqrt{x^2+1}.$$

15. The function $f(x)$ is

(a) e^{-x^2} (b) $e^{-\frac{x^2}{2}}$

(c) e^{x^2} (d) $e^{\frac{x^2}{2}}$

16. Number of solutions of $f(x) + g(x) = 0$ is

(a) 0 (b) 1 (c) 2 (d) 4

SECTION-III

MATCHING LIST TYPE (ONLY ONE OPTION CORRECT)

This section contains 4 questions, each having two matching lists. Choices for the correct combination of elements from List-I and List-II are given as options (a), (b), (c) and (d), out of which ONLY is correct.

17. Match the following:

Column I		Column II	
(P)	The value of $\sin(\sin^{-1}1)$ is	1.	-1
(Q)	The points $(k, 2-2k)$, $(-k+1, 2k)$ and $(-4-k, 6-2k)$ are collinear if k equal to	2.	1
(R)	The value of $\frac{1}{3}\left[\cos^4\left(\frac{\pi}{8}\right)+\cos^4\left(\frac{3\pi}{8}\right)+\cos^4\left(\frac{5\pi}{8}\right)+\cos^4\left(\frac{7\pi}{8}\right)\right]$ is	3.	1/2
(S)	If a, b, c are all different from zero, and $\Delta=\begin{vmatrix}1+a & 1 & 1\\ 1 & 1+b & 1\\ 1 & 1 & 1+c\end{vmatrix}$ is equal to zero then the value of $a^{-1}+b^{-1}+c^{-1}$ is	4.	$\sqrt{2}\pm1$

	P	Q	R	S
(a)	4	1, 4	2	3
(b)	2	1	2, 4	3
(c)	1, 3	3	2	4
(d)	2	1, 3	3	1

18. Match the following:

Column I		Column II	
(P)	Let $f(x) = [x-1] + [1-x]$, $[x]$ is the greatest integer function, a is an integer	1.	continuous at $x = a$
(Q)	Let f be as in (P) but a is not an integer	2.	$\lim_{x\to0} f(x)$ does not exist
(R)	Let $f(x) = \cot x$	3.	$f(a) = 0$
(S)	$f(x)=\frac{a^{\cot x}-a^{\cos x}}{\cot x-\cos x}$	4.	$\lim_{x\to\frac{\pi}{2}} f(x)=\log a$

	P	Q	R	S
(a)	4	1	2	3
(b)	2	1	4	3
(c)	1	3	2	4
(d)	3	1	2	4

19. Match the following:

Column I		Column II	
(P)	$\forall x \in R$ the value of the expression $f(x)=\sin^2 x+\sin^2\left(x+\frac{\pi}{3}\right)+\cos x\cdot\cos\left(x+\frac{\pi}{3}\right)$ is equal to	1.	1/2
(Q)	Exact value of $\cos 40°(1-2\sin 10°)$ is	2.	$-1/3$
(R)	The value of $\sum_{k=3}^{\infty}\sin^k\left(\frac{\pi}{6}\right)$, is	3.	1/4
(S)	The value of λ for which the lines are concurrent $x+y+1=0$; $3x+2\lambda y+4=0$; $x+y-3\lambda=0$ is/are	4.	5/4

	P	Q	R	S
(a)	4	3	1	2
(b)	3	4	1	2
(c)	1	3	2	4
(d)	4	1	3	2

20. Match the following:

Column I		Column II	
(P)	Let a function f is defined as $f: \{1, 2, 3, 4\} \to \{1, 2, 3, 4\}$. If f satisfy $f(f(x)) = f(x)$, $\forall\, x \in 1, 2, 3, 4$ then number of such function, is	1.	9
(Q)	If m and M are the least and greatest value of $f(x) = (\cos^{-1} x)^2 + (\sin^{-1} x)^2$ then $\frac{M}{m}$ has the value equal to	2.	10
(R)	Let x and y be two real numbers such that $2\sin x \sin y + 3\cos y + 6\cos x \sin y = 7$. The value of $\tan^2 x + 2\tan^2 y$, is	3.	1200
(S)	Number of ways in which 5 different toys can be distributed in 5 children if exactly one child does not get any toy	4.	41

	P	Q	R	S
(a)	4	2	1	3
(b)	3	4	1	2
(c)	1	3	2	4
(d)	2	4	1	3

ANSWERS

PAPER-I

1. (a, b) **2.** (b, d)

3. (b, c) **4.** (a, b, c)

5. (a, d) **6.** (a, b, c, d)

7. (c) **8.** (a, b, c)

9. (a, b, c, d) **10.** (a, d)

11. (1) **12.** (4) **13.** (1) **14.** (1) **15.** (2)
16. (6) **17.** (3) **18.** (2) **19.** (5) **20.** (3)

PAPER - II

1. (c) **2.** (d) **3.** (a) **4.** (c) **5.** (c)
6. (b) **7.** (a) **8.** (d) **9.** (a) **10.** (d)
11. (b) **12.** (c) **13.** (d) **14.** (d) **15.** (b)
16. (c) **17.** (d) **18.** (d) **19.** (d) **20.** (a)

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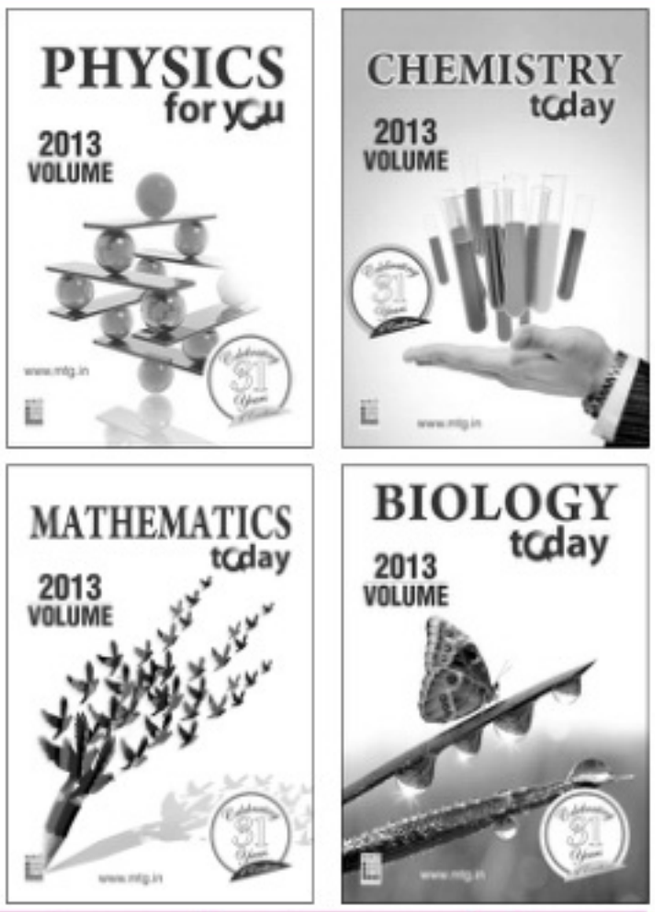

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PRACTICE PAPER 2015

JEE Main & Advanced

SECTION - I

SINGLE CORRECT CHOICE TYPE

This section contains 8 single correct choice type questions. Each question has 4 choices: (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

1. If r, s, t are prime numbers and x, y, z are positive integers such that LCM of x, y, z is $r^2s^4t^2$, then the number of possible triplets (x, y, z) is

(a) 19.61.19 (b) 20.70.20

(c) 18.60.18 (d) 21.60.21

2. The set of all values of 'a' for which $f(x) = x^3 + (a + 2)x^2 + 3ax + 5$ has a positive point of maxima, is

(a) $(-1, 4)$ (b) $(-\infty, 1)$

(c) $(4, \infty)$ (d) no value of a

3. The centre of a square $ABCD$ is z_0. If A is z_1, then centroid of ΔABC is

(a) $2z_0 - i(z_1 - z_0)$ (b) $z_0 + i\left(\frac{z_1 - z_0}{3}\right)$

(c) $\frac{z_0 + iz_1}{3}$ (d) $\frac{2}{3}(z_1 - z_0)$

4. If I is the incentre of a ΔABC and P and Q are the feet of perpendiculars from A to BI and CI, respectively, then $\frac{AP}{BI} + \frac{AQ}{CI}$ is equal to

(a) $\cot\frac{A}{2}$ (b) $\tan\frac{A}{2}$

(c) $\sin\frac{A}{2}$ (d) $\cos\frac{A}{2}$

5. Four vectors are such that they are making equal angles with each other, then angle between vectors is

(a) $\frac{\pi}{2}$ (b) $\cos^{-1}\left(\frac{1}{3}\right)$

(c) $\pi - \cos^{-1}\left(\frac{1}{3}\right)$ (d) none of these

6. If $a \in \left(0, \frac{\pi}{2}\right)$, then the integral $\int_0^a \ln(\cot a + \tan x)dx$ is

(a) 0 (b) $\ln \cos a$

(c) $-a \ln \sin a$ (d) $-\frac{a}{2}\ln \cot a$

7. Let $f : N \to N$ be a function such that $x - f(x) = 19\left[\frac{x}{19}\right] - 90\left[\frac{f(x)}{90}\right]$, $\forall x \in N$, where [.] denotes the greatest integer function and $1900 < f(1990) < 2000$, then possible value of $f(1990)$ is

(a) 1994 (b) 1924

(c) 1954 (d) 1934

8. Total number of solutions of the equation $\sin \pi x = |\log_e |x||$ is

(a) 8 (b) 10 (c) 9 (d) 6

SECTION - II

MULTIPLE CORRECT CHOICE TYPE

This section contains 4 multiple correct choice type questions. Each question has 4 choices: (a), (b), (c) and (d) for its answer, out of which ONE OR MORE is/are correct.

9. Two curves C_1 and C_2 are represented by $\arg\left(\frac{z+i}{z+1}\right) = \pm\frac{\pi}{4}$ and $\left|\frac{z+i}{z+1}\right| = 1$ respectively.

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Then which of the following is/are true?
(a) both C_1 and C_2 passes through (1, 1)
(b) curve C_2 bisects the area formed by C_1
(c) both curves are orthogonal
(d) both curves are circles

10. A square is inscribed in a circle. If p_1 is the probability that a randomly chosen point of the circle lies within the square and p_2 is the probability that the point lies outside the square, then
(a) $p_1 = p_2$ (b) $p_1 > p_2$
(c) $p_1 < p_2$ (d) $p_1^2 - p_2^2 < \frac{1}{3}$

11. In ΔABC, lengths of median AD and BE are $3x$ and $3y$ respectively. If $AD \perp BE$, then which of the following is/are true?
(a) Area of the Δ is $6xy$ square units
(b) If $\sqrt{2}x < y$, then Δ is obtuse
(c) Ratio of side AB to median from C is 2 : 3
(d) Area of quadrilateral ($DCEG$)
$= \frac{1}{6}$ area of ΔABC (G is centroid)

12. $\lim_{n\to\infty} \frac{-3n+(-1)^n}{4n-(-1)^n}$ is
(a) $-\frac{3}{4}$
(b) $-\frac{3}{4}$ if n is odd
(c) $-\frac{3}{4}$ if n is even
(d) limit does not exist if n is even

SECTION - III

COMPREHENSION TYPE

This section contains 2 paragraphs. Each paragraph has 3 multiple choice questions. Each question has 4 choices: (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

Passage-I

Two curved mirrors with equation $y = f(x)$ and $y = g(x)$ passing through (8, 6) are so placed such that all light rays emerging from the origin, after getting reflected from the mirror become parallel to x-axis.

13. Curves $f(x)$ and $g(x)$ are
(a) both parabolas
(b) one circle and one parabola
(c) both circles
(d) either both circles or both parabolas

14. Angle between tangents to $y = f(x)$ and $y = g(x)$ at (8, 6) is
(a) $\pi/3$ (b) $\pi/6$
(c) $\pi/2$ (d) $\pi/4$

15. Area included between $y = f(x)$ and $y = g(x)$ is
(a) 100 sq. units (b) 0
(c) 50 sq. units (d) 80 sq. units

Passage–II

$f(x)$ and $g(x)$ are polynomials of degree 3.

$g(\alpha) = g'(\alpha) = 0$. If $\lim_{x\to\alpha} \frac{f(x)}{g(x)} = 0$, then

16. Number of different solutions of equation $f(x)g'(x) + f'(x)g(x) = 0$ are
(a) 3 (b) 5 (c) 1 (d) 2

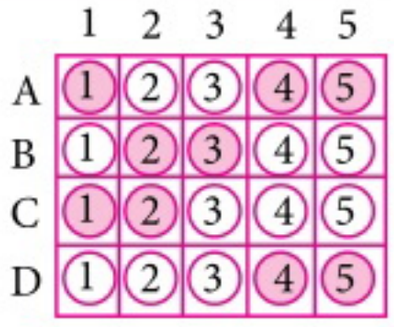
17. For the function $y = f(x)\, g(x)$ point α is
(a) point of maxima
(b) point of inflextion
(c) point of minima
(d) point of discontinuity

18. For $h > 0$, if $f(\alpha - h) < 0$, then which of the following is true?
(a) $f''(\alpha - h) > 0$ (b) $f'(\alpha - h) > 0$
(c) $f''(\alpha + h) > 0$ (d) none of these

SECTION - IV

MATRIX MATCH TYPE

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in Column I are labelled A, B, C and D, while the statements in Column II are labelled 1, 2, 3, 4 and 5. Any given statement in Column I can have correct matching with ONE OR MORE statement(s) in Column II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example: If the correct matches are A – 1, 4 and 5; B – 2 and 3; C – 1 and 2; and D – 4 and 5; then the correct darkening of bubbles will look like the given.



19. Match the following:

	Column I		Column II
(A)	Equation $x^4 + kx^2 + k = 0$ has exactly two distinct real roots. If $k \in S$, then S contains	1.	$\left(-\infty, -\frac{4}{3}\right)$
(B)	a, b, c are sides of a triangle such that $k = \frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$. If $k \in S$, then S contains	2.	$\left(-\frac{4}{3}, 0\right)$
(C)	Equation $3x + y + 4z = 3$, $x + 2y - 3z = -2$ and $6x + 5y + kz = -3$ has at least one solution. If $k \in S$, then S contains	3.	$\left(1, \frac{3}{2}\right)$
(D)	$kx^2 - 4x + (3k + 1) > 0$ for all $x > 0$, then if $k \in S$, then S contains	4.	$\left[\frac{3}{2}, 2\right)$
		5.	$[2, \infty)$

20. Let (α, β) be the coordinates of a point on the curve $(x - 2)^2 + (y - 2)^2 = 1$.

	Column I		Column II
(A)	$\frac{\max.(\alpha+\beta) - \min.(\alpha+\beta)}{2}$	1.	$\sqrt{2}$
(B)	$\max.(\alpha - \beta)$	2.	$10 + \sqrt{13}$
(C)	$\max.(\alpha\beta) - \left(\frac{\max.(\alpha+\beta)}{2}\right)^2$	3.	0
(D)	$\max.\ (2\alpha + 3\beta)$	4.	$-\min.(\alpha - \beta)$
		5.	$\max.\ (\alpha + \beta) + \max.(\alpha - \beta) + \min.(\alpha + \beta) + \min.(\alpha - \beta) - 8$

SOLUTIONS

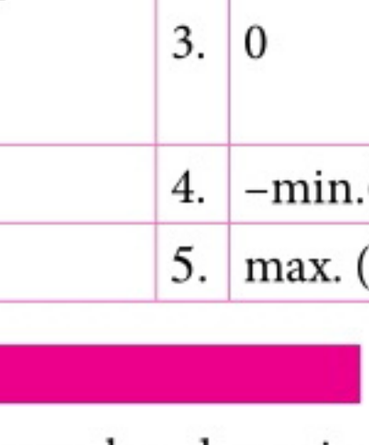
1. (a) : Selections of x, y, z can be done in $(3^3 - 2^3).(5^3 - 4^3).(3^3 - 2^3)$ ways.

2. (d) : $f'(x) = 3x^2 + 2(a + 2)x + 3a$ must have both roots positive, which is not possible.

3. (b) : $z_3 = 2z_0 - z_1$

$$\frac{z_2 - z_0}{z_1 - z_0} = e^{i\pi/2}$$

$$\Rightarrow \quad z_2 = z_0 + i(z_1 - z_0)$$



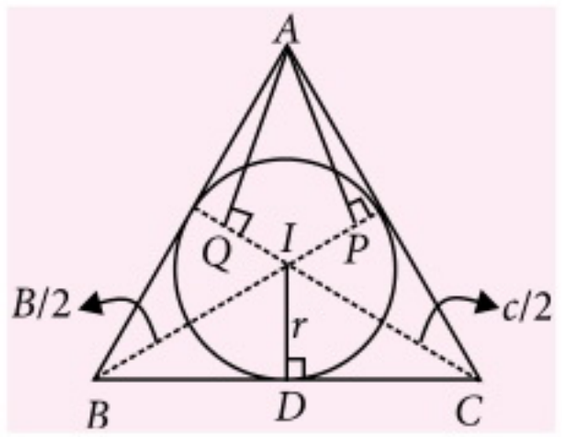
$$\text{Centroid of } \Delta ABC = \frac{z_1 + z_2 + z_3}{3}$$

4. (a) : ΔABP, $\sin\frac{B}{2} = \frac{AP}{c}$

$$\Rightarrow \quad AP = c\sin\frac{B}{2}$$

ΔAQC, $\sin\frac{C}{2} = \frac{AQ}{b}$

$$\Rightarrow \quad AQ = b\sin\frac{C}{2}$$



ΔBDI, $\sin\frac{B}{2} = \frac{r}{BI} \Rightarrow BI = \frac{r}{\sin B/2}$

Similarly, $CI = \frac{r}{\sin C/2}$

Now, $\frac{AP}{BI} + \frac{AQ}{CI} = \frac{c}{r}\sin^2\frac{B}{2} + \frac{b}{r}\sin^2\frac{C}{2}$

$$= \frac{1}{2r}[c + b - (c\cos B + b\sin C)] = \frac{b + c - a}{2r} = \frac{s - a}{r} = \cot\frac{A}{2}$$

5. (c): Vectors are lines joining centre to vertices of a regular tetrahedron.

6. (c): $I = \int_0^a \ln \frac{\cos(a-x)}{\sin a \cos x}\, dx$

$= \int_0^a \ln(\cos(a-x))dx - \int_0^a \ln(\cos x)dx - \int_0^a \ln(\sin a)dx$

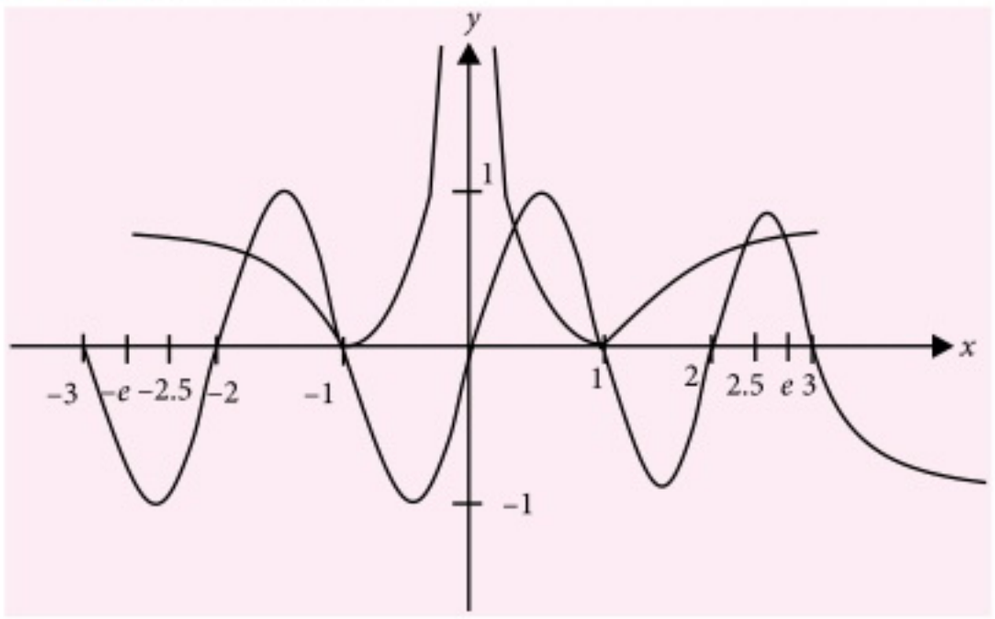
7. (a): $1990 - f(1990) = 19\left[\frac{1990}{19}\right] - 90\left[\frac{f(1990)}{90}\right]$

$\because \quad 21.11 < \frac{f(1990)}{90} < 22.22$

$\therefore \quad \left[\frac{f(1990)}{90}\right] = 21, 22$

$\Rightarrow$ $f(1990) = 1994$ or 1904

8. (d) : 6 solutions



9. (b, c) : Curve C_1 is the major circular arc of circles $x^2 + y^2 = 1$ and $(x+1)^2 + (y+1)^2 = 1$

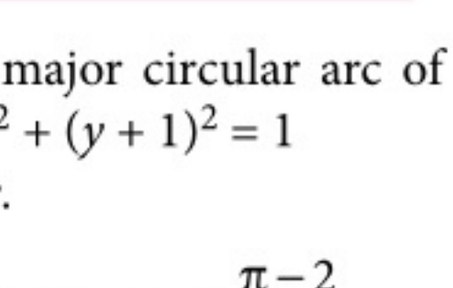
Curve C_2 is straight line $x = y$.

10. (b, d) : $p_1 = \frac{2a^2}{\pi a^2} = \frac{2}{\pi}, \quad p_2 = 1 - p_1 = \frac{\pi - 2}{\pi}$

$\Rightarrow \quad p_1^2 - p_2^2 = \frac{4-\pi}{\pi}$

11. (a, b, c, d) : $c = 2\sqrt{x^2+y^2}; a = 2\sqrt{x^2+4y^2}$

$b = 2\sqrt{4x^2+y^2}$



Area of ΔABC

$= 3$ (area of ΔAGB) $= 3\,(2xy)$

Δ is obtuse if $\cos A < 0$

i.e. $b^2 + c^2 - a^2 < 0$

Length of median from $C = \frac{1}{2}\sqrt{2a^2+2b^2-c^2}$

Area of quadrilateral $(DCEG) = \frac{1}{2}$ (area of ΔAGB)

12. (a, b, c) : If n is even

$$\lim_{n\to\infty} \frac{-3n+(-1)^n}{4n-(-1)^n} = \lim_{n\to\infty} \frac{-3n+1}{4n-1} = -\frac{3}{4}$$

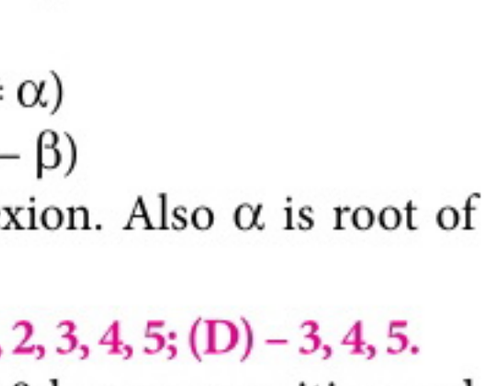
If n is odd $\lim_{n\to\infty} \frac{-3n+(-1)^n}{4n-(-1)^n} = \lim_{n\to\infty} \frac{-3n-1}{4n+1} = -\frac{3}{4}$

Passage–I

13. (a) **14. (c)** **15. (d)**

Sol.: Coordinates of N are $\left(x + y\frac{dy}{dx}, 0\right)$

Clearly, $OP = ON$



$\Rightarrow \quad \sqrt{x^2+y^2} = \left| x + y\frac{dy}{dx} \right|$

$\Rightarrow \quad \frac{d(x^2+y^2)}{2\sqrt{x^2+y^2}} = \pm dx$

$\Rightarrow \quad \sqrt{x^2+y^2} = \pm x + c$

Curves are passing through (8, 6)

So, equation of curves are $y^2 = 4(x+1)$ and $y^2 = 36(9-x)$

Passage–II

16. (d) **17. (b)** **18. (c)**

Sol.: Obviously $f(\alpha) = f'(\alpha) = f''(\alpha) = 0$

So $f(x) = \lambda(x-\alpha)^3$

$g(x) = \mu(x-\alpha)^2(x-\beta); (\beta \neq \alpha)$

$\Rightarrow \quad f(x)g(x) = \lambda\mu(x-\alpha)^5(x-\beta)$

So point α is point of inflexion. Also α is root of both $f(x) = 0$ and $f''(x) = 0$.

19. (A) – 1, 2; (B) – 4; (C) – 1, 2, 3, 4, 5; (D) – 3, 4, 5.

(A) Equation $t^2 + k.t + k = 0$ has one positive and one negative root.

$\therefore \quad f(0) < 0 \Rightarrow \quad k < 0$

(B) $\frac{a}{b+c} < \frac{2a}{a+b+c}; \frac{b}{c+a} < \frac{2b}{a+b+c}; \frac{c}{a+b} < \frac{2c}{a+b+c}$

$\Rightarrow \quad \frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} < \frac{2(a+b+c)}{(a+b+c)} = 2$

$$\frac{\left(\frac{b+c}{a}\right)^{-1} + \left(\frac{c+a}{b}\right)^{-1} + \left(\frac{a+b}{c}\right)^{-1}}{3} \geq \left(\frac{\frac{b+c}{a} + \frac{c+a}{b} + \frac{a+b}{c}}{3}\right)^{-1}$$

$\Rightarrow \quad \frac{3}{2} \leq k < 2$

(C) Equations either have unique solution or infinite solutions for all values of k except $k = \frac{1}{5}$

(D) Parabola must be upward so $k > 0$

Also, $D = -4(k-1)(3k+4)$

So when $k > 1$, $D < 0$

20. (A) – 1, 4 ; (B) – 1, 4 ; (C) – 3, 5 ; (D) – 2.

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